Varianta 025

SUBIECTUL **I**

a) 30 .
b) $\sqrt{10}$.
c) $\sqrt{17}$.
d) $\begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 2 & 5 & 1 \end{vmatrix} = 0$

e) 40.
f) $\dfrac{\sqrt{15}}{4}$.

SUBIECTUL II

1.
a) -12.
b) $\dfrac{4}{5}$.
c) 2.
d) 1. .
e) 0 .

2.
a) $3x^2+1$**.**
b) $f'(x) > 0, \forall x \in \mathbf{R}$ deci f este crescătoare $\forall x \in \mathbf{R}$.
c) 13.
d) $S = \{1, -1\}$.
e) $-\dfrac{8025}{4}$**.**

SUBIECTUL III
a)

$$xy - \frac{1}{xy} - \left(x + y - \frac{1}{x} - \frac{1}{y} \right) = \frac{(xy)^2 - 1}{xy} - \frac{x^2y + xy^2 - y - x}{xy} = \frac{(xy-1)(xy+1)}{xy} - \frac{x(xy-1) + y(xy-1)}{xy} =$$

$$= \frac{(xy-1)(x-1)(y-1)}{xy}, \ \forall x, y \in \mathbf{R}^*$$

b) Din **a) ,** $x^3-\frac{1}{x^3}-\left(x+x^2-\frac{1}{x}-\frac{1}{x^2}\right)=\frac{(x-1)(x^2-1)(x^3-1)}{x^3}$ **.**

Atunci $\frac{(x-1)(x^2-1)(x^3-1)}{x^3}=0$ **.**Deci $x=1$ sau $x=-1$.

c) Din **a)** , $ab-\frac{1}{ab}-\left(a+b-\frac{1}{a}-\frac{1}{b}\right)=\frac{(a-1)(b-1)(ab-1)}{ab}\geq 0,\forall a,b\in[1,+\infty)$.

Deci $ab-\frac{1}{ab}\geq a+b-\frac{1}{a}-\frac{1}{b},\forall a,b\in[1,+\infty)$.

d) Din **c)** , $a_1a_2-\frac{1}{a_1a_2}\geq a_1+a_2-\frac{1}{a_1}-\frac{1}{a_2}$.

Demonstrăm $P_k\Rightarrow P_{k+1}\quad \forall k\in\mathbf{N}^*$.

$P_k:\ a_1\cdot a_2\cdot\ldots\cdot a_k-\frac{1}{a_1\cdot a_2\cdot\ldots\cdot a_k}\geq a_1+a_2+\ldots+a_k-\frac{1}{a_1}-\frac{1}{a_2}-\ldots-\frac{1}{a_k}$,

adevarată $\forall\ a_1,a_2,\ldots\ldots a_k\in[1,\infty)$

Cum $a_1,a_2,\ldots\ldots a_k\in[1,\infty)\Rightarrow\ a_1\cdot a_2\cdot\ldots\ldots\cdot a_k\in[1,\infty)$ iar din **c)** avem

$(a_1a\ \ldots\ldots a_k)\cdot a_{k+1}-\frac{1}{(a_1a_2\ldots\ldots a_k)a_{k+1}}\geq a_1a_2\ldots\ldots a_k+a_{k+1}-\frac{1}{a_1a_2\ldots\ldots a_k}-\frac{1}{a_{k+1}}\geq$

$\geq a_1+a_2+\ldots\ldots+a_k+a_{k+1}-\frac{1}{a_1}-\frac{1}{a_2}-\ldots\ldots-\frac{1}{a_k}-\frac{1}{a_{k+1}}$.

Am obtinut $P_k\Rightarrow P_{k+1},\ \forall k\in\mathbf{N}^*$. Deci

$$a_1\cdot a_2\cdot\ldots\cdot a_n-\frac{1}{a_1\cdot a_2\cdot\ldots\cdot a_n}\geq a_1+a_2+\ldots+a_n-\frac{1}{a_1}-\frac{1}{a_2}-\ldots-\frac{1}{a_n}.$$

e) Cum $2^a,2^b,2^c\geq 1,\forall a,b,c\in(0,\infty)$ atunci din **d) ,** pentru $n=3$ avem că

$2^a\cdot 2^b\cdot 2^c-\frac{1}{2^a\cdot 2^b\cdot 2^c}\geq 2^a+2^b+2^c-\frac{1}{2^a}-\frac{1}{2^b}-\frac{1}{2^c}$.Deci

$2^a\cdot 2^b\cdot 2^c-2^{-a}\cdot 2^{-b}\cdot 2^{-c}\geq 2^a+2^b+2^c-2^{-a}-2^{-b}-2^{-c}$.

f)Cum $x>y>0$ rezultă $\frac{1}{x}<\frac{1}{y}$ **.** Atunci $x-\frac{1}{x}>y-\frac{1}{y}$**.**

g)Din **d) ,** pentru $a_1=a_2=\ldots=a_n$ avem $a^n-\frac{1}{a^n}\geq na-\frac{n}{a}$.

SUBIECTUL IV

a) $f_1(x)=\int_0^x f_0(t)dt=t\Big|_0^x=x,\ \forall x\in\mathbf{R}$.

b) $f_2(x)=\int_0^x f_1(t)dt=\frac{t^2}{2}\Big|_0^x=\frac{x^2}{2}$.

c) $f_1(x)+f_2(x)=0$,devine $x+\frac{x^2}{2}=0$.

Atunci $x=0$ sau $x=-2.$

d)Din **b)** , $f_2(x)=\frac{x^2}{2!}$ .Presupunem $f_k(x)=\frac{x^k}{k!}$, $\forall x\in \mathbf{R}$ şi demonstrăm

$f_{k+1}(x)=\frac{x^{k+1}}{(k+1)!}$, $\forall x\in \mathbf{R}$.

$$f_{k+1}(x)=\int_0^x \frac{t^k}{k!}dt=\int_0^x t'\cdot\frac{t^k}{k!}dt=\left(t\cdot\frac{t^k}{k!}\right)\Bigg|_0^x-\int_0^x \frac{t\cdot k\cdot t^{k-1}}{k!}dt=\frac{x^{k+1}}{k!}-k\int_0^x \frac{t^k}{k!}dt=$$

$$= \frac{x^{k+1}}{k!} - k \cdot \frac{t^{k+1}}{k!(k+1)}\bigg|_0^x = \frac{x^{k+1}}{(k+1)!},\ \forall x \in \mathbf{R}.$$

Deci $f_n(x)=\frac{x^n}{n!}, \forall x \in \mathbf{R}$.

e) $f'_{n+1}(x)=\frac{(n+1)\cdot x^n}{(n+1)!}=\frac{x^n}{n!}=f_n(x)$.

f) $\lim\limits_{x\to\infty}\frac{f_3(x)}{f_2(x)}=+\infty$

g) $\lim\limits_{n\to\infty} f_n(1)=0$.