VARIANTA 042

SUBIECTUL I

a) Coeficientul unghiular al dreptei d este 3, deci dreapta cautata este $y-3=3(x+1) \Leftrightarrow y=3x+6$

b) $d(A,d)=\frac{|3+3-2|}{\sqrt{10}}=\frac{2\sqrt{10}}{5}$

c) Evident, simetricul lui $A(-1,3)$ fata de OY este $A'(1,3)$

d) $x^2+2x+y^2=0 \Leftrightarrow (x+1)^2+y^2=1$ deci centrul este $A'(-1,0)$

e) $|a+i|=|a-1+i| \Leftrightarrow \sqrt{1+a^2}=\sqrt{(a-1)^2+1} \Leftrightarrow 1+a^2=1+a^2-2a+1 \Leftrightarrow a=\frac{1}{2}$

f) Notez $z=\left(1+i\cdot tg\frac{\pi}{3}\right)^2 \Rightarrow z=\left(1+i\sqrt{3}\right)^2=1-3+2i\sqrt{3}==-2+2i\sqrt{3} \Rightarrow$ Real $z=-2$

SUBIECTUL II

1.

a) $\log_2 4+\log_2\frac{1}{4}=\log_2 2^2+\log_2 2^{-2}=2-2=0$

b) Deoarece suma a doua pare sau doua impare este pară, probabilitatea ceruta este $\frac{2}{6}=\frac{1}{3}$

c) $x_1^2+x_2^2+x_3^2=(x_1+x_2+x_3)^2-2(x_1x_2+x_1x_3+x_2x_3)=4-2\cdot 3=-2$

d) $2^{2^x}=16 \Leftrightarrow 2^{2^x}=2^4 \Leftrightarrow 2^x=2^2 \Leftrightarrow x=2$

e) $\det A=4-6=-2$

2.

a) $f'(x)=2e^{2x}\ \forall x\in R$

b) $f''(x)=4e^{2x}>0 \Rightarrow f$ este convexă pe R

c) $f'(x)+xf(x)=0 \Leftrightarrow 2e^{2x}+xe^{2x} \Leftrightarrow x+2=0 \Leftrightarrow x=-2$

d) $\int_0^{\frac{1}{2}} f(x)\cdot f'(x)dx=\frac{1}{2}\int_0^{\frac{1}{2}}\left(f^2(x)\right)'dx=\frac{1}{2}\left(f^2(x)\right)\Big|_0^{\frac{1}{2}}=\frac{1}{2}f^2\left(\frac{1}{2}\right)-\frac{1}{2}f^2(0)=$

$=\frac{1}{2}e^{4\cdot\frac{1}{2}}-\frac{1}{2}e^{4\cdot 0}=\frac{1}{2}\left(e^2-1\right)$

e) $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty} e^{2x}=0$

SUBIECTUL III

a) $tr(A)=5.$

b) Fie $B=C=\begin{pmatrix} e & f \\ g & h \end{pmatrix}$. Atunci $tr(B)=e+h=tr(C)$.

c) Alegem $P=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ si $Q=\begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix}$ si $tr(P)=tr(Q)=5, P=Q$

d) Cum $U\cdot V=\begin{pmatrix} p & q \\ r & s \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} p & 0 \\ r & 0 \end{pmatrix}$, rezulta ca $tr(U\cdot V)=p$.

Apoi $U\cdot W=\begin{pmatrix} p & q \\ r & s \end{pmatrix}\cdot\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & p \\ 0 & r \end{pmatrix}$ şi $tr(U\cdot W)=r$.

e) Fie $D=\begin{pmatrix} d & e \\ f & g \end{pmatrix}$ si $E=\begin{pmatrix} m & n \\ p & q \end{pmatrix}$. Atunci $(aD+bE)=$

$=\begin{pmatrix} ad+bm & ae+bn \\ af+bp & ag+bq \end{pmatrix}$ si $tr(aD+bE)=ad+bm+ag+bq.$ Apoi

$a\cdot tr(D)+b\cdot tr(E)=a(d+g)+b(m+q)=tr(aD+bE).$

f) Luăm $F=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ si $G=\begin{pmatrix} x & y \\ z & t \end{pmatrix}$ si $F\cdot G=\begin{pmatrix} ax+bz & ay+bt \\ cz+dz & cy+dt \end{pmatrix}$ sau

$tr(F\cdot G)=ax+bz+cy+dt$. Apoi $G\cdot F=\begin{pmatrix} xa+yc & xb+yd \\ za+tc & zb+dt \end{pmatrix}$ si

$tr(G\cdot F)=ax+cy+bz+dt=tr(F\cdot G)$.

g) Fie $L=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ si $N=\begin{pmatrix} e & f \\ g & h \end{pmatrix}$. Daca luam $X=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, vom obtine $c=g$; pentru

$X=\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$va rezula $b=f$, iar pentru $X=\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ vom avea $d=h$. Deci $L=N$.

SUBIECTUL IV

a) $f'(x)=(\ln(x+1)-\ln x)+\left(x+\frac{1}{2}\right)\left(\frac{1}{x+1}-\frac{1}{x}\right)=\ln\frac{x+1}{x}-\frac{2x+1}{2x(x+1)}(\forall)0x>0$

b) $f''(x)=\frac{1}{x+1}-\frac{1}{x}+\frac{1}{x+1}-\frac{1}{x}+(x+\frac{1}{2})\left(-\frac{1}{(x+1)^2}+\frac{1}{x^2}\right)=$

$=\frac{2}{x+1}-\frac{2}{x}+\frac{2x+1}{2}\cdot\frac{-x^2+x^2+2x+1}{x^2(x+1)^2}=\frac{-2}{x(x+1)}+\frac{(2x+1)^2}{2x^2(x+1)^2}=$

$=\frac{-4x^2-4x+4x^2+4x+1}{2x^2(x+1)^2}=\frac{1}{2x^2(x+1)^2}$, $(\forall)\ x>0$

c) $f''(x)=\frac{1}{2x^2(x+1)^2}>0, (\forall)\ x>0 \Rightarrow f'$ este strict crescatoare pe $(0,\infty)$

d) $\lim\limits_{x\to\infty} f'(x)=\lim\limits_{x\to\infty}\left[\ln\frac{x}{x+1}-\frac{2x+1}{2x(x+1)}\right]=\ln 1-0=0$

e) $\int_1^e f'(x)dx= f(e)-f(1)=\left(e+\frac{1}{2}\right)(\ln(e+1)-1)-\frac{3}{2}(\ln 2-\ln 1)=\left(e+\frac{1}{2}\right)\ln\frac{e+1}{e}-\frac{3\ln 2}{2}$

f) Deoarece $\lim\limits_{x\to\infty} f'(x)=0$ si f' este strict crescatoare, rezulta ca $f'(x)<0\ (\forall)\ x>0 \Rightarrow f$ e strict descrescatoare.

g) Fie $n\in \mathbb{N}^*$; deoarece f este strict descrescatoare si $n<n+1\Rightarrow$

$\Rightarrow f(n)>f(n+1)\Leftrightarrow\left(n+\frac{1}{2}\right)\ln\frac{n+1}{n}>\left(n+1+\frac{1}{2}\right)\ln\frac{n+2}{n+1}\Leftrightarrow$

$\Leftrightarrow \ln\left(\frac{n+1}{n}\right)^{n+\frac{1}{2}}>\ln\left(\frac{n+2}{n+1}\right)^{n+1+\frac{1}{2}}\Leftrightarrow\left(1+\frac{1}{n}\right)^{n+\frac{1}{2}}>\left(1+\frac{1}{n+1}\right)^{n+1+\frac{1}{2}}$ (am folosit monotonia functiei ln)