VARIANTA 036

SUBIECTUL I

a) $\left|\sqrt{2}-i\sqrt{7}\right|=\sqrt{2+7}=3$

b) $AC=\sqrt{4^2+1^2}=\sqrt{17}$

c) $z=1+i^4+i^6+i^{11}=1+1+i^2+i^3=1-i \Rightarrow$ Real $z=1$

d) Formez sistemul $\begin{cases}7+2a+b=0\\11+3a+b=0\end{cases}$ cu solutia $\begin{cases}a=-4\\b=1\end{cases}$

e) Aria $[ABC]=\dfrac{|\Delta|}{2}$ unde $\Delta=\begin{vmatrix}7&2&1\\2&2&1\\11&3&1\end{vmatrix}=\begin{vmatrix}7&2&0\\2&2&0\\11&3&-2\end{vmatrix}=-2(14-4)=-20$, deci Aria$[ABC]=10$

f) $\sin^2 x+\cos^2 x=1 \Leftrightarrow \cos^2 x=1-\sin^2 x=1-\dfrac{16}{25}=\dfrac{9}{25}$

SUBIECTUL II

1.

a) $\hat{1}\cdot\hat{2}\ldots\hat{7}=\left(\hat{2}\cdot\hat{4}\right)\cdot\hat{1}\cdot\hat{3}\cdot\hat{5}\cdot\hat{6}\cdot\hat{7}=\hat{0}$

b) $C_7^3-C_7^4+C_{10}^{10}=C_7^3-C_7^3+C_{10}^{10}=1$

c) $\log_4 x=2 \Leftrightarrow x=16$

d) $16^x=32^x \Leftrightarrow 2^{4x}=2^{5x} \Leftrightarrow 4x=5x \Leftrightarrow x=0$

e) Deoarece numai 3, 4 si 5 verifică inegalitatea $5^n>30$, probabilitatea ceruta este $\dfrac{3}{5}$

2.

a) $f'(x)=5x^4+12x^2 (\forall)x\in R$

b) $\int_0^1 f(x)dx=\left(\dfrac{x^6}{6}+x^4-5x\right)\Big|_0^1=\dfrac{1}{6}+1-5=\dfrac{-23}{6}$

c) $\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x}=f'(0)=0$

d) $f'(x)=5x^4+12x^2\geq 0(\forall)x\in R \Rightarrow f$ crescătoare pe R

e) $\lim\limits_{n\to\infty}\dfrac{4\sqrt{n}+3}{7\sqrt{n}+2}=\lim\limits_{n\to\infty}\dfrac{\sqrt{n}\left(4+\dfrac{3}{\sqrt{n}}\right)}{\sqrt{n}\left(7+\dfrac{2}{\sqrt{n}}\right)}=\dfrac{4}{7}$

SUBIECTUL III

a) Fie $u(x)=ax^2+bx+c, v(x)=a'x^2+b'x+c$ cu $a,b,c,a',b',c'\in R$, $a,a'\neq 0$;

am $(u\circ v)(x)=u(v(x))=a\left(a'x^2+b'x+c\right)^2+b\left(a'x^2+b'x+c\right)+c$ şi se vede că deoarece $a\cdot a'\neq 0$, coeficientul lui x^4 este nenul adică $u\circ v\in B$

b) Fie $f\in A, f(x)=ax^2+bx+c$; $a,b,c\in R, a\neq 0$; atunci $f(x)=a\left(x+\dfrac{b}{2a}\right)^2-\dfrac{b^2-4ac}{4a}(\forall)x\in R$;

$$\left.\begin{array}{l} f\left(-x-\dfrac{b}{2a}\right)=ax^2-\dfrac{b^2-4ac}{4a} \\ f\left(-\dfrac{b}{2a}+x\right)=ax^2-\dfrac{b^2-4ac}{4a} \end{array}\right\} \text{ de unde } f\left(-x-\dfrac{b}{2a}\right)=f\left(-\dfrac{b}{2a}+x\right)(\forall)x\in R$$

c) $s(x)=x^4=(x^2)^2$ şi notez $u(x)=v(x)=x^2$ şi am $u,v\in A$ şi $(u\circ v)(x)=s(x)$,
deci $s\in C$

d) Fie $g(x)=(x-1)^4\in B;$ am $g(1-x)=x^4, g(1+x)=x^4$ deci $g(1-x)=g(1+x)(\forall)x\in R$

e) Presupun că $(\exists)a\in R$ cu $h(a-x)=h(a+x)(\forall)x\in R$; atunci h(0)=h(2a) sau
$1=16a^4+2a+1\Leftrightarrow 2a(8a^3+1)=0\Leftrightarrow a=0$ sau $a=-\dfrac{1}{2}$. Daca a=0, rezultă $h(-1)=h(x),$ fals căci $h(1)\neq h(-1)$.
Daca $a=-\dfrac{1}{2}\Rightarrow h\left(-\dfrac{1}{2}-x\right)=h\left(-\dfrac{1}{2}+x\right)$ adică $\left(x+\dfrac{1}{2}\right)^4-\left(x-\dfrac{1}{2}\right)^4=2x(\forall)x\in R,$ fals căci $\left(\dfrac{3}{2}\right)^4-\left(\dfrac{1}{2}\right)^4\neq 2$

f) Fie $x\in C;$ atunci $w=u\circ v; u,v\in A$ şi cum $(\exists)c\in R$ astfel încât
$v(c-x)=v(c+x)(\forall)x\in R\Rightarrow (u\circ v)(c-x)=(u\circ v)(c+x)(\forall)x\in R$ deci $w(c-x)=w(x+c)(\forall)x\in R$

g) $h(x)=x^4+x+1;$ h evident $h\in B, h\notin C$ (din f), deci $B\setminus C\neq\phi$

SUBIECTUL IV

a) $F(x)=1+\int_0^x f(t)dt=1+\left(t+t^2+t^3+t^4\right)\Big|_0^x=1+x+x^2+x^3+x^4(\forall)x\in R$

b) $F'(x)=\left(1+x+x^2+x^3+x^4\right)'=1+2x+3x^2+4x^3=f(x),(\forall)x\in R$

c) $(x-1)F(x)=(x-1)(x^4+x^3+x^2+x+1)=x^5-1,(\forall)x\in R$

d) Pentru x=1, $F(1)=5>0$ iar pentru $(\forall)x\in R, x\neq 1$ am $F(x)=\dfrac{x^5-1}{x-1}>0;$
Concluzie: $F(x)>0(\forall)x\in R$

e) $F''(x)=2+6x+12x^2=2\left(6x^2+3x+1\right)>0,(\forall)x\in R$ (căci $\Delta_x=9-24<0$) deci F este convexă pe R

f) $\lim\limits_{n\to\infty}\dfrac{nf(n)}{F(n)}=\lim\limits_{n\to\infty}\dfrac{4n^4+3n^3+2n^2+n}{n^4+n^3+n^2+n+1}=4$

g) $f(x^3)=1\Leftrightarrow 4x^9+3x^6+2x^3+1=1\Leftrightarrow x^3(4x^6+3x^3+2)=0$ cu x=0 soluție unică în R