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1. Other solutions for the problem 202 and the problem 204 from La Gaceta de la RSME

by Roxana – Mihaela Stanciu[1] and Nela Ciceu[2]

Abstract. *The solutions of the problem 202 and the problem 204 was presented in La Gaceta de la RSME, Vol. 16 (2013), No.2, pp. 289-292. Here we present new solutions for this two problems.*

PROBLEMA 202. *Propuesto por Panagiote Ligouras, "Leonardo da Vinci" High School, Noci, Italia.*

Para un triángulo ABC denotaremos por r su inradio, por r_a, r_b y r_c sus exinradios, por I su incentro, y por I_a, I_b e I_c sus exincentros. Probar o refutar la desigualdad

$$\frac{\cos A}{1-\cos^2 A}+\frac{\cos B}{1-\cos^2 B}+\frac{\cos C}{1-\cos^2 C}\geq \frac{1}{4r}\sqrt{\frac{II_a\cdot II_b\cdot II_c}{r_ar_br_c}(r_ar_b+r_br_c+r_cr_a)}.$$

Solution:

Since we well-known that:

$$II_a=4R\sin\frac{A}{2}\ ,\ \frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}=\frac{1}{r}\text{ and }r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2},$$

We deduce that:

$$\frac{II_a\cdot II_b\cdot II_c}{r_ar_br_c}(r_ar_b+r_br_c+r_cr_a)=\frac{64R^3\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{r}=16R^2,$$

And we have to prove that:

$$\frac{\cos A}{\sin^2 A}+\frac{\cos B}{\sin^2 B}+\frac{\cos C}{\sin^2 C}\geq\frac{R}{r}\Leftrightarrow(1)\frac{\cos A}{4R^2\sin^2 A}+\frac{\cos B}{4R^2\sin^2 B}+\frac{\cos C}{4R^2\sin^2 C}\geq\frac{1}{4Rr}$$

$$\Leftrightarrow\frac{\cos A}{a^2}+\frac{\cos B}{b^2}+\frac{\cos C}{c^2}\geq\frac{s}{abc}\Leftrightarrow(2)\frac{b^2+c^2-a^2}{2a^2bc}+\frac{c^2+a^2-b^2}{2ab^2c}+\frac{a^2+b^2-c^2}{2abc^2}\geq\frac{s}{abc}$$

$$\Leftrightarrow\frac{b^2+c^2}{a}-a+\frac{c^2+a^2}{b}-b+\frac{a^2+b^2}{c}-c\geq 2s\Leftrightarrow\frac{b^2+c^2}{a}+\frac{c^2+a^2}{b}+\frac{a^2+b^2}{c}\geq 4s\quad(*)$$

We deduce (1) by the law of sines and (2) by the law of cosines.

By *Bergström'* s inequality we deduce that:

$$\frac{b^2}{a}+\frac{c^2}{b}+\frac{a^2}{c}\geq\frac{(a+b+c)^2}{a+b+c}=2s\text{ and }\frac{c^2}{a}+\frac{a^2}{b}+\frac{b^2}{c}\geq\frac{(a+b+c)^2}{a+b+c}=2s,$$

which by adding yields to (*) and we are done.

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We have equality if and only if $a = b = c$.

PROBLEMA 204. *Propuesto por Juan Bosco Romero Márquez, Universidad Complutense de Madrid, Madrid.*

Sea ABC un triángulo y, con las notaciones usuales, definimos la cantidad

$$d = rr_a + r_b r_c - 2m_a h_a.$$

Establecer condiciones suficientes sobre los ángulos del triángulo ABC para que la cantidad d sea, respectivamente, positiva, negativa o nula.

Solution :

We denote by $S = area(ABC)$ and $p = semiperimeter(ABC)$, and by well-known formuls we obtain that:

$$d = \frac{S^2}{p(p-a)} + \frac{S^2}{(p-b)(p-c)} - 2m_a h_a = \frac{S^2(p^2 - pb - pc + bc + p^2 - pa)}{p(p-a)(p-b)(p-c)} - 2m_a h_a =$$

$$= 2p^2 - p(a+b+c) + bc - 2m_a h_a = bc - 2m_a h_a = \frac{2S}{\sin A} - 2m_a \cdot \frac{2S}{a} = \frac{2S}{a}\left(\frac{a}{\sin A} - 2m_a\right),$$

i.e.

$$d = \frac{4S}{a}\left(R - m_a\right).$$

To compare d with 0 returns to study the sign of the expresion $R - m_a$ according to the angles of the triangle ABC, and this is the subject of the problem 3113 from CRUX MATHEMATICORUM, with solution in no. 1/2007, pp. 63-64 (see the solution from the pages below).We are done.

3113. [2006 : 47, 49; 171, 174] *Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valladolid, Spain.*

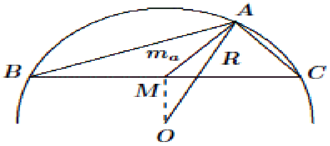
Let ABC be a triangle and let a be the length of the side opposite the vertex A. If m_a is the length of the median from A to BC, and if R is the circumradius of $\triangle ABC$, prove that $m_a - R$ is positive, negative, or zero, according as $\angle A$ is obtuse, acute, or right-angled.

Combination of similar solutions by Roy Barbara, University of Beirut, Beirut, Lebanon; and Richard I. Hess, Rancho Palos Verdes, CA, USA.

We let M be the mid-point of BC and consider the triangle OMA with sides $AM = m_a$ and $AO = R$. According to Euclid, the relative sizes of these two sides depends on the size of the opposite angles.

Case 1. A is obtuse.

Vertex A (on the circumcircle) is separated from the circumcentre O by the chord BC. Since $OM \perp BC$, $\angle OMA$ is obtuse; whence, the opposite side R is longer than the adjacent side m_a; that is, $m_a - R < 0$ when A is obtuse, as claimed.



Case 2. $A = 90°$.

Here BC is a diameter; thus, $m_a = R$, and $m_a - R = 0$.

2. O întărire a problemei L:298 din Sclipirea Minţii XI/2013

de Nela Ciceu[3] și Roxana Mihaela Stanciu[4]

Problema în discuţie, propusă de prof. *Ovidiu Ţâţan* din Râmnicu Sărat, are următorul enunţ:

''Fie x_1, x_2, x_3 rădăcinile ecuaţiei $x^3 - 10x^2 + 28x - 18 = 0$. Să se arate că $\sqrt{x_1} + \sqrt[3]{x_2} + \sqrt[4]{x_3} > 3$.''

Vom demonstra o întărire a inegalităţii cerute.

Fie $f(x) = x^3 - 10x^2 + 28x - 18$. Folosind schema lui Horner sau prin calcul direct, avem:

x	0,5	1	3,5	4	5,5	6
f(x)	-6,375	1	0,375	-2	-0,125	6

Deci ecuaţia dată aretrei rădăcini reale positive, situate în intervalele

$$\left(\frac{1}{2},1\right), \left(\frac{7}{2},4\right) \text{ și } \left(\frac{11}{2},6\right).$$

În primul tabel de mai jos avem cu aproximaţie prin lipsă, rădăcinile de ordin 2, 3 și 4 ale numerelor 0,5; 3,5 și 5,5.

x	$\sqrt{x}$	$\sqrt[3]{x}$	$\sqrt[4]{x}$
0,5	0,70	0,79	0,84
3,5	1,87	1,52	1,36
5,5	2,34	1,76	1,53

În al doilea tabel avem, cu aproximaţie prin adaos, rădăcinile de ordin 2, 3 și 4 ale numerelor 1, 4 și 6.

x	$\sqrt{x}$	$\sqrt[3]{x}$	$\sqrt[4]{x}$
1	1	1	1
4	2	1,59	1,42
6	2,45	1,82	1,57

Rezultă că

$$\sqrt{x_1} + \sqrt[3]{x_2} + \sqrt[4]{x_3} > \min(3{,}75; 3{,}82; 4{,}19; 4{,}49; 4{,}47; 4{,}7) = 3{,}75;$$
$$\sqrt{x_1} + \sqrt[3]{x_2} + \sqrt[4]{x_3} < \max(4{,}16; 4{,}24; \ 4{,}57; 4{,}87; 4{,}82; 5{,}04) = 5{,}04.$$

Deci

$$3{,}75 < \sqrt{x_1} + \sqrt[3]{x_2} + \sqrt[4]{x_3} < 5{,}04.$$

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Observaţie. Considerând pentru rădăcinile ecuaţiei date intervalele

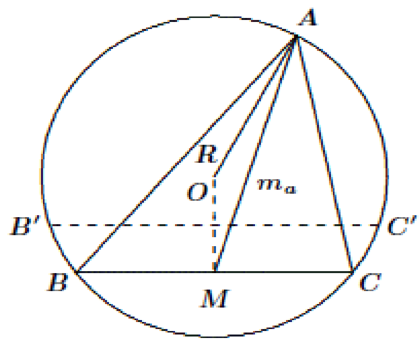
$$(0{,}91;1),\ (3{,}55;3{,}6) \text{ și } (5{,}5;5{,}6)$$

obţinem

$$4<\sqrt{x_1}+\sqrt[3]{x_2}+\sqrt[4]{x_3}<4{,}91.$$

Case 3. $\boldsymbol{A}$ is acute.

The proposal is incorrect: $m_a - R$ can be positive, zero, or negative when A is acute, as follows. Let $B'C'$ be the perpendicular bisector of OM. For A on the long arc of the circumcircle between B' and C', we have $\angle MOA > \angle OMA$; whence $m_a - R > 0$. For A at B' or at C', we get $m_a - R = 0$. Finally, when A lies on either short arc $B'B$ or $C'C$, we see that $m_a - R < 0$. Note that the proposal becomes correct for triangles ABC in which *all* angles are acute; then A will necessarily lie on the arc $B'C'$ which, as we have just seen, forces $m_a - R > 0$, as claimed in the proposal.



*Also solved by MICHEL BATAILLE, Rouen, France; CHIP CURTIS, Missouri Southern State University, Joplin, MO, USA; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; GEOFFREY A. KANDALL, Hamden, CT, USA; *VEDULA N. MURTY, Dover, PA, USA; *PETER Y. WOO, Biola University, La Mirada, CA, USA; LI ZHOU, Polk Community College, Winter Haven, FL, USA; TITU ZVONARU, Comănești, Romania (with two proofs for the obtuse-angle case); and the *proposer. The asterisk designates solutions that were correct, but whose analysis of the acute-angle case was incomplete. In addition VÁCLAV KONEČNÝ, Big Rapids, MI, USA provided a counterexample showing that the conclusion to the corrected proposal still was flawed. There were three incorrect submissions.*

3. THE SOLUTIONS OF SOME PROBLEMS OF KAPPA MU EPSILON

IOAN VIOREL CODREANU, SECONDARY SCHOOL SATULUNG, MARAMUREȘ

PROBLEM 699. ***Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA.***

1. ***Find all positive integers*** x ***such that*** $2^x+2^{11}+2^8$ ***is a perfect square.***
2. ***Find all positive integers*** x ***such that*** $4^x+4^{11}+4^8$ ***is a perfect square.***

Solution. 1. We have

$$2^x+2^{11}+2^8=2^x+2^8\left(2^3+1\right)=2^x+2^8\cdot 9=2^x+\left(2^4\cdot 3\right)^2=2^x+48^2.$$

Let $t\in N$ so that

$$2^x+48^2=t^2\Leftrightarrow (t-48)(t+48)=2^x$$

whence we get $t-48=2^\alpha$ and $t+48=2^\beta$ with $\alpha<\beta$ and $\alpha+\beta=x.$

We have

$$2^\beta-2^\alpha=96\Leftrightarrow 2^\alpha\left(2^{\beta-\alpha}-1\right)=2^5\cdot 3$$

and we get $\alpha=5$ and $\beta-\alpha=2$, namely $\beta=7$. Then $t=2^5+48=80$ and

$$2^x=(80-48)(80+48)\Leftrightarrow 2^x=32\cdot 128\Leftrightarrow 2^x=2^{12}\Leftrightarrow x=12.$$

2. We have $4^x+4^{11}+4^8=2^{2x}+4^8\left(4^3+1\right)=2^{2x}+4^8\cdot 65=2^{2x}+2^{16}\cdot 65.$

Let $y\in N$ so that

$$2^{2x}+2^{16}\cdot 65=y^2\Leftrightarrow \left(y-2^x\right)\left(y+2^x\right)=2^{16}\cdot 5\cdot 13.$$

For $x=0$ we have $4^0+4^{11}+4^8=4259841$ it isn′t a perfect square. For $x\geq 1, y$ is even.

We studied the cases:

$$(i)\begin{cases} y-2^x=2^\alpha\cdot 5\cdot 13\\ y+2^x=2^\beta\cdot 5\cdot 13\end{cases}$$

with $\alpha+\beta=16,\alpha<\beta,1\leq\alpha,\beta\leq 15$. We have $2^{x+1}=2^\alpha\cdot 5\cdot 13\left(2^{\beta-\alpha}-1\right)$, impossible.

$$(ii)\begin{cases} y-2^x=2^\lambda\cdot 5\\ y+2^x=2^\delta\cdot 13\end{cases}$$

with $\lambda+\delta=16,1\leq\lambda,\delta\leq 15.$ We have $2^{x+1}=2^\delta\cdot 13-2^\lambda\cdot 5$.

If $\delta > \lambda$, then $2^{x+1} = 2^{\lambda}\left(2^{\delta-\lambda} \cdot 13 - 5\right)$, impossible because 2^{x+1} is even and $2^{\delta-\gamma} \cdot 13 - 5$ is odd.

If $\delta = \lambda = 8$, then $2^{x+1} = 2^{11} \Leftrightarrow x = 10$.

If $\delta < \lambda$, then $2^{x+1} = 2^{\delta}\left(13 - 5 \cdot 2^{\lambda-\delta}\right)$.

If $\lambda - \delta \geq 2$, then $2^{x+1} = 2^{\delta}\left(13 - 5 \cdot 2^{\lambda-\delta}\right) \leq 2^{\delta}(13-20) < 0$, false.

If $\lambda - \delta = 1$, then $2^{x+1} = 2^{\delta} \cdot 3$, impossible. So, $x = 10$.

PROBLEM 703. ***Proposed by Pedro H.O. Pantoja (student), University of Natal, Brazil.***

Let $x, y,$ ***and*** z ***be positive real numbers. Prove that***

$$\left(x^3+y^3+z^3+3xyz\right)^{2(x+y+z)} \geq [2xy(x+y+z)]^{x+y} \cdot [2yz(x+y+z)]^{y+z} \cdot [2xz(x+y+z)]^{x+z}$$

Solution. Using the **Weighted AM-GM Inequality**

$$\sum_{i=1}^{n} w_i a_i \geq \prod_{i=1}^{n} a_i^{w_i}, a_i, w_i \geq 0, \sum_{i=1}^{n} w_i = 1, \text{ for}$$

$$n = 3, a_1 = 2xy(x+y+z), a_2 = 2yz(x+y+z), a_3 = 2xz(x+y+z), w_1 = \frac{x+y}{2(x+y+z)},$$

$$w_2 = \frac{y+z}{2(x+y+z)}, w_3 = \frac{x+z}{2(x+y+z)}$$

and the **Schur Inequality**

$$x^3+y^3+z^3 \geq xy(x+y)+yz(y+z)+xz(x+z)$$

we have

$$\prod\left(2xy(x+y+z)^{\frac{x+y}{2(x+y+z)}}\right) \leq \sum\left(\frac{x+y}{2(x+y+z)} \cdot 2xy(x+y+z)\right) = \sum xy(x+y) \leq \sum x^3 + 3\prod x$$

whence we get

$$\left(\sum x^3 + 3\prod x\right)^{2\sum x} \geq \prod\left(\left(2xy\sum x\right)^{x+y}\right).$$

PROBLEM 706. ***Proposed by Lose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.***

Let x ***be a positive real numbers. Prove that***

$$\frac{x+\{x\}}{[x]^2+2\{x\}^2}+\frac{x+[x]}{\{x\}^2+2[x]^2}<\frac{4}{x},$$

where $[x]$ ***and*** $\{x\}$ ***are the integer and fractional part of*** x***, respectively.***

Solution. We note $a=[x], b=\{x\}$. Then $x=a+b$ and the inequality of the statement is written equivalent

$$\frac{(a+2b)(a+b)}{a^2+2b^2}+\frac{(2a+b)(a+b)}{2a^2+b^2}<4 \Leftrightarrow$$

$$\frac{a^2+2b^2+3ab}{a^2+2b^2}+\frac{2a^2+b^2+3ab}{2a^2+b^2}<4 \Leftrightarrow$$

$$1+\frac{3ab}{a^2+2b^2}+1+\frac{3ab}{2a^2+b^2}<4 \Leftrightarrow$$

$$\frac{3ab}{a^2+2b^2}+\frac{3ab}{2a^2+b^2}<2 \Leftrightarrow$$

$$\frac{9ab\left(a^2+b^2\right)}{\left(a^2+2b^2\right)\left(2a^2+b^2\right)}<2 \Leftrightarrow$$

$$9a^3b+9ab^3<4a^4+10a^2b^2+4b^4 \Leftrightarrow$$

$$4\left(a^2-b^2\right)^2-9ab(a-b)^2>0 \Leftrightarrow$$

$$(a-b)^2\left(4(a+b)^2-9ab\right)>0$$

the last inequality is true because $4(a+b)^2>16ab\geq 9ab$ and $(a-b)^2>0$, if we account of $a\neq b$.

PROBLEM 707. ***Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.***

Let n ***be an odd positive integer and*** p ***a prime number of the form*** $3n+2$***. Prove that if***

$$\frac{a}{b}=\sum_{i=1}^{2n+1}\frac{(-1)^{i+1}}{i}$$

then p ***divides*** a***.***

Solution. Using the **Catalan Identity**

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{2n-1}-\frac{1}{2n}=\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}$$

we get

$$\frac{a}{b}=\sum_{i=1}^{2n+1}\frac{(-1)^{i+1}}{i}=\sum_{i=1}^{n+1}\frac{1}{n+i}$$

and by grouping together the terms of the sum, we have

$$\frac{a}{b}=\left(\frac{1}{n+1}+\frac{1}{2n+1}\right)+\left(\frac{1}{n+2}+\frac{1}{2n}\right)+\ldots$$

and

$$\frac{a}{b}=\frac{3n+2}{(n+1)(2n+1)}+\frac{3n+2}{(n+2)2n}+\ldots$$

whence it follows that it exists the positive integer number q so that

$$\frac{a}{b}=\frac{pq}{(n+1)(n+2)\cdot\ldots\cdot(2n+1)}$$

and

$$pqb=a(n+1)(n+2)\cdot\ldots\cdot(2n+1)$$

whence it follows that

$$p\mid a(n+1)(n+2)\cdot\ldots\cdot(2n+1)$$

and how $(p,(n+1)(n+2)\cdot\ldots\cdot(2n+1))=1$ it follows that $p\mid a$.

Problem 694. ***Proposed by Jose Luis Diaz – Barrero, BARCELONA TECH, Barcelona, Spain***

Let a,b,c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a}\geq\frac{1}{6}$$

***First solution*:**

By the **Cauchy – Schwarz Inequality** we have

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a}=\frac{a^4}{a(a+b)}+\frac{b^4}{b(b+c)}+\frac{c^4}{c(c+a)}\geq\frac{\left(a^2+b^2+c^2\right)^2}{a^2+b^2+c^2+ab+bc+ca}.$$

Using the inequality $a^2+b^2+c^2 \ge ab+bc+ca$ (the inequality is equivalent to $(a-b)^2+(b-c)^2+(c-a)^2 \ge 0$), we get

$$\frac{a^3}{a+b}+\frac{b^3}{b+a}+\frac{c^3}{c+a} \ge \frac{\left(a^2+b^2+c^2\right)^2}{2\left(a^2+b^2+c^2\right)} = \frac{a^2+b^2+c^2}{2}.$$

Using the **Cauchy – Schwarz Inequality** we get

$$a^2+b^2+c^2 \ge \frac{(a+b+c)^2}{3} = \frac{1}{3}.$$

So

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a} \ge \frac{1}{6}.$$

***Second solution*:**

By the **Cauchy – Schwartz Inequality** we have

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a} \ge \frac{\left(a\sqrt{a}\right)^2}{a+b}+\frac{\left(b\sqrt{b}\right)^2}{b+c}+\frac{\left(c\sqrt{c}\right)^2}{c+a}$$
$$\ge \frac{\left(a\sqrt{a}+b\sqrt{b}+c\sqrt{c}\right)^2}{2(a+b+c)} = \frac{\left(a\sqrt{a}+b\sqrt{b}+c\sqrt{c}\right)^2}{2}.$$

Now, consider the function $f:(0,1)\to(0,\infty),\ f(x)=x\sqrt{x}$. A short computation of derivatives shows f is convex, $f''(x)=\frac{3}{4\sqrt{x}}>0, \forall x\in(0,1)$.

Applying **Jensen′s Inequality** we get

$$a\sqrt{a}+b\sqrt{b}+c\sqrt{c} \ge 3\cdot\frac{a+b+c}{3}\cdot\sqrt{\frac{a+b+c}{3}} = \frac{1}{\sqrt{3}}.$$

So

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a} \ge \frac{1}{6}.$$

***Third solution*:**

Using the **AM-GM Inequality**, we deduce that

$$\frac{a^3}{a+b}+\frac{a+b}{12} \ge 2\sqrt{\frac{a^3}{a+b}\cdot\frac{a+b}{12}} = \frac{a\sqrt{a}}{\sqrt{3}}$$

Similarly, $\frac{b^3}{b+c}+\frac{b+c}{12}\geq\frac{b\sqrt{b}}{\sqrt{3}}$ and $\frac{c^3}{c+a}+\frac{c+a}{12}\geq\frac{c\sqrt{c}}{\sqrt{3}}$. Adding these three inequalities, we have

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a}+\frac{2(a+b+c)}{12}\geq\frac{a\sqrt{a}+b\sqrt{b}+c\sqrt{c}}{\sqrt{3}}.$$

Then

$$\frac{a^3}{a+b}+\frac{b^3}{b+c}+\frac{c^3}{c+a}\geq\frac{a\sqrt{a}+b\sqrt{b}+c\sqrt{c}}{\sqrt{3}}-\frac{1}{6}\geq\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$$

because $a\sqrt{a}+b\sqrt{b}+c\sqrt{c}\geq\frac{1}{\sqrt{3}}$ (***Second solution***).

Problem 690. ***Proposed by Tom Moore, Bridegewater State University, Bridgewater, MA***

Show that the sequence 91, 8911, 889111, 88891111, … (i.e. , put one more 8 on the front and on more 1 on the end) consists solely of triangular numbers.

Solution.

Let $a_n=\underbrace{88...8}_{n}9\underbrace{11...1}_{n+1}$, T_n the n^{th} triangular number, n natural number. We have

$$\begin{aligned}
a_n &= 1+10+10^2+...+10^n+9\cdot10^{n+1}+8\cdot\left(10^{n+2}+10^{n+3}+...+10^{2n+1}\right)\\
&=\frac{10^{n+1}-1}{10-1}+9\cdot10^{n+1}+8\cdot\frac{10^{2n+2}-10^{n+2}}{10-1}\\
&=\frac{8\cdot10^{2n+2}+2\cdot10^{n+1}-1}{9}\\
&=\frac{\left(4\cdot10^{n+1}-1\right)\left(2\cdot10^{n+1}+1\right)}{9}\\
&=\frac{\frac{4\cdot10^{n+1}-1}{3}\cdot\frac{4\cdot10^{n+1}+2}{3}}{2}\\
&=\frac{\frac{4\cdot10^{n+1}-1}{3}\cdot\left(\frac{4\cdot10^{n+1}-1}{3}+1\right)}{2}\\
&=T_{\frac{4\cdot10^{n+1}-1}{3}}\\
&=T_{1\underbrace{33...3}_{n+1}}
\end{aligned}$$

and the proof is complete.

Problem 691. ***Proposed by Tom Moore, Bridegewater State University, Bridgewater, MA***

Which pentagonal numbers are the positive integer power of a single prime ?

Solution.

Let P_n the n^{th} pentagonal number, $n \geq 1$. It is known that $P_n = \frac{n(3n-1)}{2}$. Let p prime number, k natural number, $k \geq 1$, such that $\frac{n(3n-1)}{2} = p^k$, equivalent to the equality

$$n(3n-1) = 2p^k \qquad (1)$$

We distinguish two cases:

Case I. If n is even, then there are natural numbers m such that $n = 2m, m \geq 1$.

The equality (1) becomes

$$2m(6m-1) = 2p^k$$

equivalent to

$$m(6m-1) = p^k$$

from which we deduce that there are natural numbers u, v such that $m = p^u$ and $6m-1 = p^v$,

$u + v = k$ and $u < v$. Substituting $m = p^u$ in $6m-1 = p^v$ we get

$$6p^u - p^v = 1 \qquad (2)$$

If $u \geq 1$ of (2) we have $p\left(6p^{u-1} - p^{v-1}\right) = 1$ from which we deduce that p is divisor to 1, absurd. If $u = 0$ of (2) we get $p^v = 5$ and then $p = 5, v = 1, m = 1$ and $n = 2$.

Case II. If n is odd, then there are natural numbers m such that $n = 2m+1, m \geq 1$.

The equality (1) becomes

$$(2m+1)(6m+2) = 2p^k$$

equivalent to

$$(2m+1)(3m+1) = p^k$$

from which we deduce that there are natural numbers u, v such that

$2m+1 = p^u, 3m+1 = p^v$,

$u+v=k$ and $u<v$. Substituting $m=\frac{p^u-1}{2}$ in $3m+1=p^v$ we get

$$3p^u - 2p^v = 1.$$

If $v \geq u+1$, then $1 = 3p^u - 2p^v \leq 3p^u - 2p^{u+1} = p^u(3-2p) < 0$, absurd. So $v < u+1$, but then $u < v < u+1$, absurd. Finally the solution is $n=2, P_2=5$.

Problem 693. ***Proposed by D.M. Bătinețu-Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania***

Prove that if $x, y, m > 0$, then in any triangle ABC, the following holds

$$\frac{\tan^{2m+1}\frac{A}{2}}{\left(x\cot\frac{B}{2}+y\cot\frac{C}{2}\right)^m}+\frac{\tan^{2m+1}\frac{B}{2}}{\left(x\cot\frac{C}{2}+y\cot\frac{A}{2}\right)^m}+\frac{\tan^{2m+1}\frac{C}{2}}{\left(x\cot\frac{A}{2}+y\cot\frac{B}{2}\right)^m}\geq\frac{(4R+r)r^m}{(x+y)^m s^{m+1}}$$

where R is the circumradius, r is the inradius, and s is the semiperimeter.

Solution.

Lemma. If $p \geq 1, 0 \leq r \leq p-1, a_i \geq 0$ ***and*** $b_i > 0, i=\overline{1,n}$***, then:***

$$\sum_{i=1}^{n}\frac{a_i^{\,p}}{b_i^{\,r}}\geq\frac{\left(\sum_{i=1}^{n}a_i\right)^p}{n^{p-r-1}\cdot\left(\sum_{i=1}^{n}b_i\right)^r} \qquad \textbf{(1)}$$

Proof. For $r=0$ we have

$$\sum_{i=1}^{n}a_i^{\,p}\geq\frac{\left(\sum_{i=1}^{n}a_i\right)^p}{n^{p-1}}\Leftrightarrow\frac{\sum_{i=1}^{n}a_i^{\,p}}{n}\geq\left(\frac{\sum_{i=1}^{n}a_i}{n}\right)^p\Leftrightarrow\left(\frac{\sum_{i=1}^{n}a_i^{\,p}}{n}\right)^{\frac{1}{p}}\geq\frac{\sum_{i=1}^{n}a_i}{n}$$

is true because $p \geq 1$.

For $p=1$ is obtained $r=0$ and $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} a_i$ is true.

For $p>1$ and $0<r\le p-1$ apply **Hölder's Inequality** numbers $\frac{a_i}{b_i^{\frac{r}{p}}}$ and $b_i^{\frac{r}{p}}, i=\overline{1,n}$,

with

$p>1$ and $q>0$ such that $\frac{1}{p}+\frac{1}{q}=1$ and obtain

$$\left(\sum_{i=1}^{n}\left(\frac{a_i}{b_i^{\frac{r}{p}}}\right)^p\right)^{\frac{1}{p}} \cdot \left(\sum_{i=1}^{n}\left(b_i^{\frac{r}{p}}\right)^q\right)^{\frac{1}{q}} \ge \sum_{i=1}^{n} a_i .$$

Raising the last inequality to the power p and using $q=\frac{p}{p-1}$, we get

$$\sum_{i=1}^{n}\frac{a_i^{\,p}}{b_i^{\,r}} \ge \frac{\left(\sum_{i=1}^{n} a_i\right)^p}{\left(\sum_{i=1}^{n} b_i^{\frac{r}{p-1}}\right)^{p-1}} \qquad (2)$$

For $p>1$ and $0<r\le p-1$ we have $\frac{p-1}{r}\ge 1$ and then:

$$\frac{\sum_{i=1}^{n} b_i^{\frac{r}{p-1}}}{n} \le \left(\frac{\sum_{i=1}^{n}\left(b_i^{\frac{r}{p-1}}\right)^{\frac{p-1}{r}}}{n}\right)^{\frac{r}{p-1}},$$

from which we deduce:

$$\left(\sum_{i=1}^{n} b_i^{\frac{r}{p-1}}\right)^{p-1} \le n^{p-r-1}\cdot\left(\sum_{i=1}^{n} b_i\right)^r \qquad (3)$$

Using (2) and (3) we get (1).

Using *Lemma* for

$$n=3,\ p=2m+1, r=m, a_1=\tan\frac{A}{2}, a_2=\tan\frac{B}{2}, a_3=\tan\frac{C}{2}, b_1=xctg\frac{B}{2}+yctg\frac{C}{2},$$

$$b_2=xctg\frac{C}{2}+yctg\frac{A}{2}, b_3=xctg\frac{A}{2}+yctg\frac{B}{2}$$

we have

$$\sum\frac{\tan^{2m+1}\frac{A}{2}}{\left(xctg\frac{B}{2}+yctg\frac{C}{2}\right)^m}\geq\frac{\left(\sum\tan\frac{A}{2}\right)^{2m+1}}{3^m\left((x+y)\sum ctg\frac{A}{2}\right)^m}$$

$$=\frac{\left(\frac{4R+r}{s}\right)^{2m+1}}{3^m\left((x+y)\cdot\frac{s}{r}\right)^m}=\frac{(4R+r)^{2m+1}\cdot r^m}{3^m(x+y)^m s^{3m+1}}$$

where we used the identities known $\sum\tan\frac{A}{2}=\frac{4R+r}{s}$ and $\sum ctg\frac{A}{2}=\frac{s}{r}$.

It suffices to prove that

$$\frac{(4R+r)^{2m+1}\cdot r^m}{3^m(x+y)^m s^{3m+1}}\geq\frac{(4R+r)r^m}{(x+y)^m s^{m+1}}\Leftrightarrow(4R+r)^{2m}\geq 3^m s^{2m}\Leftrightarrow 3s^2\leq 16R^2+8Rr+r^2.$$

Using **Gerretsen's Inequality** $s^2\leq 4R^2+4Rr+3r^2$ we deduce that

$3s^2\leq 12R^2+12Rr+9r^2$.

It suffices to prove that the inequality $12R^2+12Rr+9r^2\leq 16R^2+8Rr+9r^2$ equivalent to

$4Rr+8r^2\leq 4R^2$ and $Rr+2r^2\leq R^2$ we write $(R+r)(R-2r)\geq 0$ evident if we consider **Euler's Inequality** $R\geq 2r$.

Problem 682. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a,b,c three positive numbers such that $a^2+b^2+c^2=1$. Prove that

$$\left[\frac{1}{a^3(b+c)^5}+\frac{1}{b^3(c+a)^5}+\frac{1}{c^3(a+b)^5}\right]^{1/5}\geq\frac{3}{2}$$

Solution. Applying the AM – GM Inequality we get

$$\sum \frac{1}{a^3(b+c)^5} \geq 3\sqrt[3]{\prod \frac{1}{a^3(b+c)^5}} = \frac{3}{\left(\prod a\right)\left(\prod (b+c)\right)\sqrt[3]{\prod (b+c)^2}}$$

Using the AM – GM Inequality we have

$$\sqrt[3]{\prod (b+c)^2} \leq \frac{\sum (b+c)^2}{3} = \frac{2\left(\sum a^2 + \sum bc\right)}{3} = \frac{2\left(1+\sum bc\right)}{3}$$

Then

$$\sum \frac{1}{a^3(b+c)^5} \geq \frac{9}{2\left(\prod a\right)\left(\prod (b+c)\right)\left(1+\sum bc\right)}$$

And now major denominator factors. We have

$$\prod (b+c) \leq \left(\frac{2\sum a}{3}\right)^3$$

but

$$\left(\sum a\right)^2 \leq 3\left(\sum a^2\right) = 3$$

namely

$$\sum a \leq \sqrt{3}.$$

Then

$$\prod (b+c) \leq \frac{8\sqrt{3}}{9}$$

We have

$$\prod a \leq \left(\frac{\sum a}{3}\right)^3 \leq \frac{\sqrt{3}}{9}$$

and

$$\sum bc \leq \sum a^2 = 1$$

Finally

$$\sum \frac{1}{a^3(b+c)^5} \geq \frac{9}{\frac{\sqrt{3}}{9} \cdot \frac{8\sqrt{3}}{9} \cdot 2 \cdot (1+1)} = \left(\frac{3}{5}\right)^5$$

The inequality of the statement concluded practical demonstration.

Problem 681. *Proposed by Jose Luis Diaz Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain*

Let a, b, c be the lengths of the sides of an acute triangle ABC. Prove that

$$\sum_{cyclic}\left(\cos^a B \cos^b A\right)^{1/(a+b)} < 2 .$$

Solution. The inequality becomes

$$\sum_{cyclic}\left(\cos^a B \cos^b A\right)^{\frac{1}{a+b}} < 2 \Leftrightarrow \sum_{cyclic}\left(\cos^{\frac{a}{a+b}} B \cos^{\frac{b}{a+b}} A\right) < 2$$

Using the Weighted AM – GM Inequality, we have

$$\cos^{\frac{a}{a+b}} B \cos^{\frac{b}{a+b}} A \le \frac{a}{a+b}\cos B + \frac{b}{a+b}\cos A = \frac{a\cos B + b\cos A}{a+b} =$$
$$= \frac{2R(\sin A\cos B + \sin B\cos A)}{a+b} = \frac{2R\sin(A+B)}{a+b} = \frac{2R\sin C}{a+b} = \frac{c}{a+b}$$

Then

$$\sum_{cyclic}\left(\cos^{\frac{a}{a+b}} B \cos^{\frac{b}{a+b}} A\right) \le \sum_{cyclic}\frac{c}{a+b}$$

Using the known equality

$$\sum_{cyclic}\frac{c}{a+b} = \frac{2\left(p^2 - r^2 - Rr\right)}{p^2 + r^2 + Rr}$$

and the obvious inequality

$$p^2 - r^2 - Rr < p^2 + r^2 + Rr$$

we get

$$\sum_{cyclic}\frac{c}{a+b} < 2$$

and the proof is complete.

4.NUMĂRUL PI

„Iubirea este precum numărul PI: reală, iraţională şi infinit de necesară Universului”

Prof. Nistor Gabriela

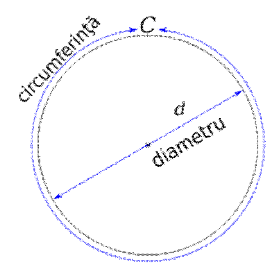
LITERA π

Numele literei grecești π este *pi*, scriere utilizată în unele situații în care nu este disponibil simbolul grecesc, sau în care utilizarea sa ar fi problematică. π corespunde literei române (latine) p. Nu se notează cu literă mare (Π) nici măcar la început de propoziție.

Constanta se numește „π” deoarece este prima literă a cuvintelor grecești περιφέρεια (*perifereia* = periferie) şi περίμετρος (*perimetros* = perimetru), cu referire la utilizarea sa în formula de calcul a circumferinței (sau a perimetrului) unui cerc.

DEFINIŢIE

Numărul Pi este o constantă matematică a cărei valoare este egală cu raportul dintre circumferinţa şi diametrul oricărui cerc într-un spaţiu euclidian, sau cu raportul dintre aria unui cerc si pătratul razei sale. Numarul pi este un număr iraţional, a carui valoare este egală, în varianta scurtă, cu **3,14**.



$$\pi = \frac{C}{d}. \quad \pi = \frac{A}{r^2}.$$

ISTORIA NUMĂRULUI PI

2550-2500 Î.H- Cea mai veche utilizare atestată a unei bune aproximări a lungimii unei circumferințe în raport cu raza este 3+1/7, valoare folosită la proiectele piramidelor din Vechiul Regat al Egiptului. Marea Piramidă din Giza, a fost construită cu un perimetru de 1.760 cubiți și o înălțime de 280 cubiți; raportul 1.760/280 ≈ 2π.

2000 Î.Hr. – Egiptenii socoteau numărul pi ca fiind $16^2 / 9^2$ sau 256/81, sau 3.16 cu o exactitate de o singură zecimală.

250 Î. Hr. – Filozoful grec Arhimede a fost primul care a estimat cât mai riguros numărul pi. El a realizat ca amploarea acestuia poate fi limitată înscriind cercuri în poligoane regulate şi calculand perimetrele externe şi interne a acestor poligoane. Folosind 96 astfel de poligoane, el a demonstrat ca **3** 10/71 < ∏ < **3** 10/70, adică 3.14185 – o exactitate de 3 zecimale.

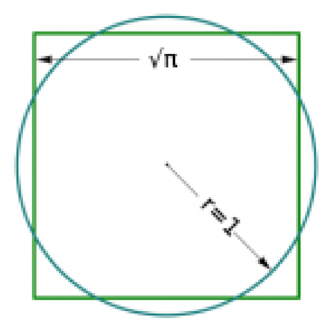
Secolul al 16-lea – Ludolph van Ceulen din Germania a calculat numarul pi cu o exactitate de 35 de zecimale dar a murit înainte de a fi publicată descoperirea. Aceasta a fost inscripţionată pe piatra lui funerară.

1706 – Astronomul englez John Machin a descoperit o formulă complicată pentru aflarea cât mai exactă a numărului pi, şi a calculat cu exactitate primele 100 de zecimale.

1873 – Matematicianul englez William Shanks s-a chinuit timp de 15 ani pentru calcularea a 707 zecimale dar din pacăte a facut o greşeală la a 528-a zecimală, rezultând ca celelalte zecimale să fie greşite la rândul lor.

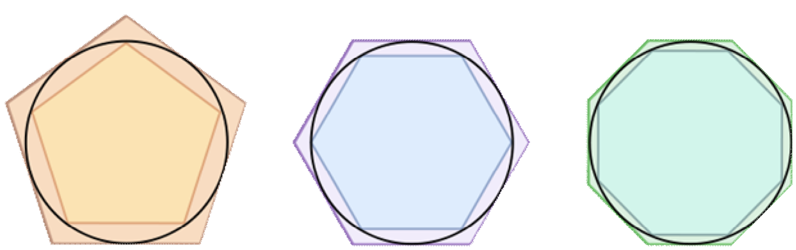
2004 – Yasumasa Kanada din Tokyo a calculat cu ajutorul unui computer un număr de 1.24 trilioane de zecimale a numarului pi.

PROPRIETĂŢI ALE NUMĂRULUI PI



- este iraţional (nu poate fi scris ca raport a două numere întregi) – iraţionalitatea sa a fost demostrată complet abia în secolul 18;
- este transcendent (nu există nici un polinom cu coeficienţi raţionali care să-l aibă pe pi ca radacină),
- nu este construibil geometric (nu se poate construi cu rigla şi compasul un pătrat cu aria egală cu cea a unui cerc dat – aceasta este o problemă de geometrie veche şi celebră, cunoscută sub numele de “Cuadratura cercului“;
- are un număr infinit de zecimale care nu conţin secvenţe ce se repetă; acest şir infinit de cifre a fascinat numeroşi matematicieni, iar în ultimele secole s-au depus eforturi semnificative pentru a investiga proprietăţile acestui număr; totuşi, în ciuda muncii analitice şi a calculelor realizate pe supercalculatoare care au calculat 10 mii de miliarde de cifre ale lui pi, nu s-a descoperit nici un şablon identificabil în cifrele găsite. Cifrele numarului pi sunt disponibile pe multe pagini web şi există programe software pentru calcularea lui pi cu miliarde de cifre precizie.

STUDIEREA NUMĂRULUI PI



Arhimede (287–212 î.e.n.) a fost primul care a încercat să calculeze valoarea lui π cu rigurozitate. El și-a dat seama că această mărime poate fi limitată superior și inferior înscriind cercurile în poligoane regulate și calculând perimetrul poligoanelor exterioare și respectiv interioare.

Folosind echivalentul unui poligon cu 96 de laturi, el a demonstrat că $3 + 10/71 < \pi < 3 + 1/7$. Media acestor valori este aproximativ 3,14185.

METODE DE APROXIMARE A NUMĂRULUI PI

Realizată de François Viète (1593)

$$\frac{\pi}{2}=\frac{1}{\sqrt{\frac{1}{2}}\times\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\times\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\times\cdots}$$

pi=3.141592653288
... după16 iterări

Realizată de Leonhard Euler

$$\pi=\frac{1}{1}+\frac{1\cdot1}{1\cdot3}+\frac{1\cdot1\cdot2}{1\cdot3\cdot5}+\frac{1\cdot1\cdot2\cdot3}{1\cdot3\cdot5\cdot7}+\cdots$$

pi=3.1415797... după 16 iterări;

Realizată de John Wallis (1655)

$$\frac{\pi}{2}=\frac{2\cdot2\cdot4\cdot4\cdot6\cdot6\cdot8\cdot8\cdot\ldots}{1\cdot3\cdot3\cdot5\cdot5\cdot7\cdot7\cdot9\cdot\ldots}$$

foarte lentă: pi=3.050... după 16 iterări

$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots$$

extrem de lentă : pi=3.079... după 16 iterări , şi este nevoie de 136.121 iterări pentru a avea 4 şi 5 zecimale, şi 2.886.751 pentru 6

Realizată de Leibniz:

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{(-1)^n}{2n+1}+\cdots=\frac{\pi}{4}$$

din care rezultă că:

$$\frac{1}{1\cdot3}+\frac{1}{5\cdot7}+\frac{1}{9\cdot11}+\frac{1}{13\cdot15}+\frac{1}{17\cdot19}+\cdots=\frac{\pi}{8}.$$

Realizată de Nilakantha

$$\frac{1}{1\cdot2\cdot3}-\frac{1}{2\cdot3\cdot5}+\frac{1}{3\cdot4\cdot7}-\frac{1}{4\cdot5\cdot9}+\ldots=\pi-3$$

Realizată de Madhava (1400)

$$\pi=\sqrt{12}\left(1-\frac{1}{3\cdot3}+\frac{1}{3^2\cdot5}-\frac{1}{3^3\cdot7}+\cdots\right)$$

Realizată de Wallis ca produs:

$$\prod_{n=1}^{\infty} \frac{(2n)(2n)}{(2n-1)(2n+1)} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdots = \frac{\pi}{2}$$

Realizată de Wallis ca produs de numere prime

$$\frac{1}{\left(1-\frac{1}{2^2}\right)} \frac{1}{\left(1-\frac{1}{3^2}\right)} \frac{1}{\left(1-\frac{1}{5^2}\right)} \frac{1}{\left(1-\frac{1}{7^2}\right)} \frac{1}{\left(1-\frac{1}{11^2}\right)} \cdots = \frac{\pi^2}{6}$$

Produsul infinit de Euler cu numere prime impare:

$$\frac{\pi}{4} = \frac{3}{4} \times \frac{5}{4} \times \frac{7}{8} \times \frac{11}{12} \times \frac{13}{12} \times \frac{17}{16} \times \frac{19}{20} \times \frac{23}{24} \times \frac{29}{28} \times \frac{31}{32} \times \cdots$$

în cazul în care numărătorul sunt toate numerele prime impare și numitorul este multiplu de patru cel mai apropiat de numărător.

Problema de la Basel :

$$\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

soluționate de către Euler .

O altă formulă care utilizează numărul Pi este funcția Riemann Zeta :

$$\zeta(4) = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots = \frac{\pi^4}{90}$$

NUMĂRUL PI EXPRIMAT CA INTEGRALĂ

Integrala lui Gauss :

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Integrala lui Euler

$$\int_{-\infty}^{+\infty} e^{\frac{-u^2}{2}} du = \sqrt{2\pi}$$

Integrala lui Fresnel

$$\int_{-\infty}^{+\infty} \cos(x^2)\, dx = \int_{-\infty}^{+\infty} \operatorname{sen}(x^2)\, dx = \sqrt{\frac{\pi}{2}}$$

Următoarele integrale definite:

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx = \pi$$

$$\int_0^{+\infty} \frac{x}{e^x - 1}\,dx = \frac{\pi^2}{6}$$

REPREZENTĂRI ALE LUI π CA FRACŢII CONTINUE :

$$\frac{4}{\pi} = 1 + \cfrac{1}{3 + \cfrac{4}{5 + \cfrac{9}{7 + \cfrac{16}{9 + \cfrac{25}{11 + \cfrac{36}{13 + \dots}}}}}}$$

$$\frac{4}{\pi} = 1 + \cfrac{1}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \cfrac{11^2}{2 + \dots}}}}}}$$

Fracția continuă a Ramanujan :

$$\sqrt{\phi^2 + 1} = \phi + \cfrac{e^{-2\pi/5}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \dots}}}}$$

în cazul în care ϕ este raportul de aur ($1,618\ldots$).

ALTE UTILIZĂRI ALE LUI π

Funcţia Euler de identitate :

$$e^{\pi i} + 1 = 0$$

Funcția gamma calculate $1/2$:

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Formulă bazată pe seria armonică , cu "corecție" de semne (Euler , 1748)

$$\pi = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} + \frac{1}{12} - \frac{1}{13} + \cdots$$

Teorema reziduurilor :

$$\oint \frac{dz}{z} = 2\pi i$$

Teorema reziduurilor :

$$\oint \frac{dz}{z} = 2\pi i$$

Alte formule de aproximare sunt prezentate în tabelul de mai jos:

$\pi = \frac{1}{Z}$	$Z = \sum_{n=0}^{\infty} \frac{((2n)!)^3(42n+5)}{(n!)^6 16^{3n+1}}$
$\pi = \frac{4}{Z}$	$Z = \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!(21460n+1123)}{(n!)^4 441^{2n+1} 2^{10n+1}}$
$\pi = \frac{4}{Z}$	$Z = \sum_{n=0}^{\infty} \frac{(6n+1)\left(\frac{1}{2}\right)_n^3}{4^n(n!)^3}$
$\pi = \frac{32}{Z}$	$Z = \sum_{n=0}^{\infty} \left(\frac{\sqrt{5}-1}{2}\right)^{8n} \frac{(42n\sqrt{5}+30n+5\sqrt{5}-1)\left(\frac{1}{2}\right)_n^3}{64^n(n!)^3}$
$\pi = \frac{27}{4Z}$	$Z = \sum_{n=0}^{\infty} \left(\frac{2}{27}\right)^n \frac{(15n+2)\left(\frac{1}{2}\right)_n\left(\frac{1}{3}\right)_n\left(\frac{2}{3}\right)_n}{(n!)^3}$
$\pi = \frac{15\sqrt{3}}{2Z}$	$Z = \sum_{n=0}^{\infty} \left(\frac{4}{125}\right)^n \frac{(33n+4)\left(\frac{1}{2}\right)_n\left(\frac{1}{3}\right)_n\left(\frac{2}{3}\right)_n}{(n!)^3}$
$\pi = \frac{5\sqrt{5}}{2\sqrt{3}Z}$	$Z = \sum_{n=0}^{\infty} \left(\frac{4}{125}\right)^n \frac{(11n+1)\left(\frac{1}{2}\right)_n\left(\frac{1}{6}\right)_n\left(\frac{5}{6}\right)_n}{(n!)^3}$
$\pi = \frac{2\sqrt{3}}{Z}$	$Z = \sum_{n=0}^{\infty} \frac{(8n+1)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 9^n}$
$\pi = \frac{\sqrt{3}}{9Z}$	$Z = \sum_{n=0}^{\infty} \frac{(40n+3)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 49^{2n+1}}$
$\pi = \frac{4\sqrt{5}}{5Z}$	$Z = \sum_{n=0}^{\infty} \frac{(644n+41)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 5^n 72^{2n-1}}$
$\pi = \frac{4\sqrt{3}}{3Z}$	$Z = \sum_{n=0}^{\infty} \frac{(-1)^n(28n+3)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 3^n 4^{n+1}}$

$\pi = \frac{4}{Z}$	$Z = \sum_{n=0}^{\infty} \frac{(-1)^n(20n+3)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 2^{2n+1}}$
$\pi = \frac{72}{Z}$	$Z = \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!(260n+23)}{(n!)^4 4^{4n} 18^{2n}}$

ZIUA MONDIALĂ A LUI PI

Pe 14 martie, comunitatea mondială a pasionaţilor de matematică şi ştiinţe va sărbători numărul pi. Celebrată în mod logic la această dată (3/14, formatul american al datei), ziua pi le dă ocazia iubitorilor ştiinţelor exacte să dea curs liber pasiunii lor pentru valoarea 3,14.

În secţiile de ştiinţe exacte din universităţi, ziua este sărbătorită prin câteva manifestări şi, tot cu această ocazie, fast-food-urile pregătesc faimoasele plăcinte 'Pi Pie', scrie Agerpres. Universitatea americană Princetown a organizat un week-end special dedicat valorii pi. Şi alte manifestări legate de pi vor avea loc, printre care proiecţia primului film al lui Darren Aronofsky, 'Pi' (1997), sau piesa cu acelaşi nume a cântăreţei Kate Bush.

"Universul este un cerc al cărui centru e pretutindeni, iar circumferinţa nicăieri."

Blaise Pascal