REVISTA ELECTRONICĂ MATEINFO.RO

ISSN 2065-6432

DECEMBRIE 2014

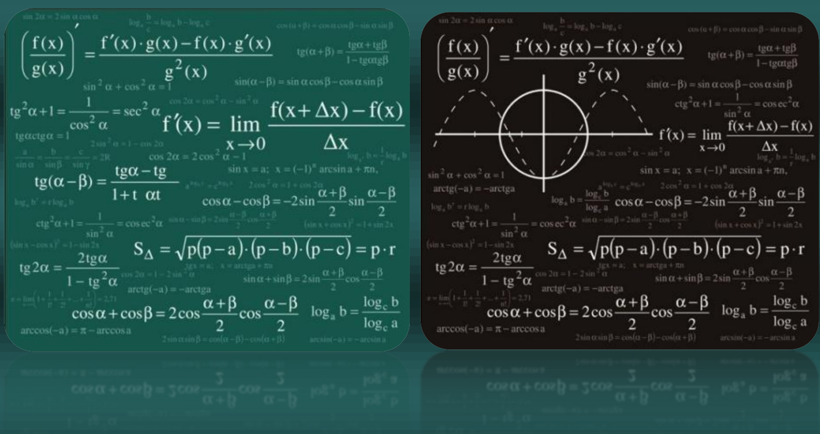
REVISTĂ LUNARĂ

DIN FEBRUARIE 2009

DE PESTE 4 ANI ÎN FIECARE LUNĂ

WWW.MATEINFO.RO

revistaelectronica@mateinfo.ro



COORDONATOR: ANDREI OCTAVIAN DOBRE

REDACTORI PRINCIPALI ŞI SUSŢINĂTOR PERMANENŢI AI REVISTEI

NECULAI STANCIU, ROXANA MIHAELA STANCIU ŞI NELA CICEU

Articole:

1. Solutins and hints of some problems from the Octogon Mathematical Magazine (IV) – pag. 2
   D.M. Bătineţu-Giurgiu, Neculai Stanciu
2. Other solutions from some problems from School Science and Mathematics journal – pag. 8
   Nela Ciceu, Roxana Mihaela Stanciu
3. Exerciţii cu progresii aritmetice. Generalizări - pag. 11
   Ciobîcă C. Constantin, Ciobîcă Elena
4. Metode de integrare numeric:
   polinomul de interpolare Lagrange, formula lui Simpson, aplicaţii - pag 19
   Boer Elena Milena

1. Solutions and hints of some problems from the Octogon Mathematical Magazine (IV)

by D.M. Bătineţu-Giurgiu, Bucharest, Romania

and

Neculai Stanciu, Buzău, Romania

PP. 20660. Solve the following system:

$$\sqrt{x_1-3}+\sqrt{x_2^2-4x_2+3}-\sqrt{(x_3-2)^3}=\sqrt{x_2-1}+\sqrt{x_3^2-4x_3+3}-\sqrt{(x_4-2)^3}=\ldots$$
$$=\sqrt{x_n-3}+\sqrt{x_1^2-4x_1+3}-\sqrt{(x_2-3)^3}=0.$$

Solution. Adding the equations of the system we obtain:

$$\sum_{k=1}^{n}\left(\sqrt{x_k-3}+\sqrt{x_k^2-4x_k+3}-\sqrt{(x_k-2)^3}\right)=0.$$

We prove that for any $x\geq 3$ we have the inequality:

$$\sqrt{x-3}+\sqrt{x^2-4x+3}\leq\sqrt{(x-3)^3}\ ,\ (1)$$

Denoting $y=x-2$, the inequality (1) becomes:

$\sqrt{y-1}+\sqrt{y^2-1}\leq\sqrt{y^3}$, which after some algebra becomes:

$\left((y-1)(y^2-1)-1\right)^2\geq 0$, true, with equality if and only if

$(y^2-1)(y-1)=1\Leftrightarrow y^3-y^2-y=0$

$\Leftrightarrow y_1=0,\ y_{2,3}=\dfrac{1\pm\sqrt{5}}{2}$. But only $y=\dfrac{1+\sqrt{5}}{2}>1$, i.e. $x=\dfrac{5+\sqrt{5}}{2}$.

So, the system has the unique solution $\left(\dfrac{5+\sqrt{5}}{2},\dfrac{5+\sqrt{5}}{2},\ldots,\dfrac{5+\sqrt{5}}{2}\right)$, and we are done.

PP.20661. In all acute triangle ABC holds $\left(\sum tgA\right)\left(\sum ctgA\right)\geq 4\left(\sum\sin A\right)\left(\sum\cos A\right)$.

Solution. The inequality from the statement is not true, for e.g. if triangle ABC is equilateral we should have $9\geq 9\sqrt{3}$.

We will prove that

$$\sqrt{3}\left(\sum tgA\right)\left(\sum ctgA\right)\geq 9\sqrt{3}\geq 4\left(\sum\sin A\right)\left(\sum\cos A\right).$$

Indeed by Bottema we have

$$\sum \sin A \leq \frac{3\sqrt{3}}{2} \text{ (the item 2.1), } \sum \cos A \leq \frac{3}{2} \text{ (the item 2.16),}$$
$$\sum tgA \geq 3\sqrt{3} \text{ (true in all acute triangle) and } \sum ctgA \geq \sqrt{3} \text{ (the item 2.38).}$$

Therefore, we get

$$\sqrt{3}\left(\sum tgA\right)\left(\sum ctgA\right) \geq 9\sqrt{3} \geq 4\left(\sum \sin A\right)\left(\sum \cos A\right), \text{ and we are done.}$$

PP. 20668. If $x, y > 0$, then:

$$x^{n-1} + x^{n-2}y + ... + xy^{n-2} + y^{n-1} \geq ((n-1)x + y)\sqrt[n]{xy^{n-1}} \geq nxy.$$

Solution. The right inequality yields by AM-GM inequality. Indeed

$$((n-1)x + y)\sqrt[n]{xy^{n-1}} = \left(\underbrace{x + x + ... + x}_{n-1} + y\right)\sqrt[n]{xy^{n-1}} \geq n \cdot \sqrt[n]{x^{n-1}y} \cdot \sqrt[n]{xy^{n-1}} = nxy.$$

The left inequality is not true. For e.g. if we take $n = 4$, $x = \frac{1}{3}$, $y = 1$ we should have

$$x^3 + x^2y + xy^2 + y^3 \geq (3x + y) \cdot \sqrt[4]{xy^3} \Leftrightarrow \frac{1}{27} + \frac{1}{9} + \frac{1}{3} + 1 \geq 2 \cdot \sqrt[4]{\frac{1}{3}}$$

$$\Leftrightarrow \frac{40}{27} \geq 2 \cdot \sqrt[4]{\frac{1}{3}} \Leftrightarrow 20 \cdot \sqrt[4]{3} \geq 27 \Leftrightarrow 400\sqrt{3} \geq 729, \text{ but } 400\sqrt{3} < 400 \cdot \frac{7}{4} = 700.$$

PP.20669. If $x_k > 0$ $(k = 1,2,...,n)$, then $\sum_{cyclic} \frac{1}{x_1 x_2^2}\left(\frac{x_1^2 + x_1x_2 + x_2^2}{2x_1 + x_2}\right)^3 \geq n$.

Solution. By AM-GM inequality $x_1 x_2^2 \leq \left(\frac{x_1 + 2x_2}{3}\right)^3$ and the inequality

$$\frac{a^2 + ab + b^2}{(a + 2b)(b + 2a)} \geq \frac{1}{3} \Leftrightarrow (a - b)^2 \geq 0,$$

yields that:

$$\sum_{cyclic} \frac{1}{x_1 x_2^2}\left(\frac{x_1^2 + x_1x_2 + x_2^2}{2x_1 + x_2}\right)^3 \geq \sum_{cyclic}\left(\frac{3(x_1^2 + x_1x_2 + x_2^2)}{(x_1 + 2x_2)(2x_1 + x_2)}\right)^3 \geq \sum_{cyclic} 1 = n,$$

and we are done.

PP.20670. If $x_k > 0$ $(k = 1,2,...,n)$, then $\sum_{cyclic} \frac{(x_1^2 + x_1x_2 + x_2^2)^2}{(2x_1 + x_2)(x_1 + 2x_2)} \geq \sum_{cyclic} x_1 x_2$.

Solution. By

$$\frac{a^2+ab+b^2}{(a+2b)(b+2a)} \geq \frac{1}{3} \Leftrightarrow (a-b)^2 \geq 0,$$

and the fact that $a^2+ab+b^2 \geq 3ab$ we obtain

$$\frac{(a^2+ab+b^2)^2}{(a+2b)(2a+b)} = (a^2+ab+b^2)\cdot\frac{a^2+ab+b^2}{(a+2b)(2a+b)} \geq 3ab\cdot\frac{1}{3} = ab,$$

and then by adding cyclic yields the conclusion, and we are done.

PP.20671. Solve in R the following system:

$$\begin{cases} x^2+xy+y^2 = (2x+y)\sqrt[3]{xz^2} \\ y^2+yz+z^2 = (2y+z)\sqrt[3]{yx^2} \\ z^2+zx+z^2 = (2z+x)\sqrt[3]{zy^2} \end{cases}.$$

Solution. We solve the system in R_+. By AM-GM inequality we have:

$$\sqrt[3]{xz^2} \leq \frac{x+2z}{3}, \text{ and other two similar.}$$

Adding the equations of the system yields:

$$2\sum x^2 + \sum xy = (2x+y)\sqrt[3]{xz^2} + (2y+z)\sqrt[3]{yx^2} + (2z+x)\sqrt[3]{zy^2} \leq$$

$$\leq \frac{(2x+y)(x+2z)}{3} + \frac{(2y+z)(y+2x)}{3} + \frac{(2z+x)(z+2y)}{3} =$$

$$= \frac{2\sum x^2 + 7\sum xy}{3}, \text{ i.e.}$$

$$6\sum x^2 + 3\sum xy \leq 2\sum x^2 + 7\sum xy \Leftrightarrow \sum x^2 \leq \sum xy, \text{ but } \sum x^2 \geq \sum xy.$$

Therefore, $x=y=z$, and we obtain the solutions (a,a,a), with $a \in R_+$.

PP.20672. If $x,y>0$ then

$$\left(1+\frac{x}{y}\right)^2\left(1+\frac{y}{x}\right)^2 \geq \left(1+\frac{2x+y}{\sqrt[3]{x^2y}}\right)\left(1+\frac{2y+x}{\sqrt[3]{xy^2}}\right) \geq 16.$$

Solution. We prove the right inequality with AM-GM inequality, i.e. we have

$$2x+y \geq 3\cdot\sqrt[3]{x^2y},\ 2y+x \geq 3\cdot\sqrt[3]{xy^2}.$$

So,

$$\left(1+\frac{2x+y}{\sqrt[3]{x^2y}}\right)\left(1+\frac{2y+x}{\sqrt[3]{xy^2}}\right)\geq(1+3)(1+3)=16.$$

After some algebraic manipulations the left inequality is equivalent with:

$$\left(\sqrt[3]{x^4}-\sqrt[3]{y^4}\right)\left(\frac{\sqrt[3]{x^8}-\sqrt[3]{y^8}}{\sqrt[3]{x^5y^5}}\right)+\left(\sqrt[3]{x^4}-\sqrt[3]{y^4}\right)\cdot 2\cdot\frac{\sqrt[3]{x^2}-\sqrt[3]{y^2}}{\sqrt[3]{x^2y^2}}\geq 0,$$ which is true

because the expressions $\sqrt[3]{x^4}-\sqrt[3]{y^4}$ and $\sqrt[3]{x^8}-\sqrt[3]{y^8}$, respectively $\sqrt[3]{x^4}-\sqrt[3]{y^4}$ and $\sqrt[3]{x^2}-\sqrt[3]{y^2}$ has the same sign.

We have equality if and only if $x=y$.

PP.20677. In all nonisosceles triangle holds

$$\left(\sum\frac{h_a-h_b}{h_c}\right)\left(\sum\frac{h_c}{h_a-h_b}\right)=\frac{(4R+r)\left((4R+r)^2-s^2\right)}{s^2r}-6.$$

Solution. The RHS is not correct. A solution and the correction for the RHS is given by M. Bencze and is the following

$$\left(\sum\frac{h_a-h_b}{h_c}\right)\left(\sum\frac{h_c}{h_a-h_b}\right)=$$

$$=\frac{48R^3+16s^2Rr(s^2+r^2+4Rr)-(s^2+r^2+4Rr)^3-48s^2R^2r^2}{16s^2R^2r^2}.$$

(a proof by M. Bencze, can be found in math journal Sclipirea Minţii – Vol. 6, No. 11, 2013, p. 9).

In fact, also in math journal Sclipirea Minţii – Vol. 6, No. 11, 2013, p. 9, was given by Bencze a proof for this identity

$$\left(\sum\frac{r_a-r_b}{r_c}\right)\left(\sum\frac{r_c}{r_a-r_b}\right)=\frac{(4R+r)\left((4R+r)^2-s^2\right)}{s^2r}-6,$$

which holds in all nonisosceles triangle

PP.20678. In all nonisosceles triangle holds

$$\left(\sum\frac{\sin(A-B)\sin^2 C}{\cos C}\right)\left(\sum\frac{\cos C}{\sin(A-B)\sin^2 C}\right)=$$

$$=6-\frac{(s^2-4Rr-r^2)(4s^2r^2-(s^2-r^2-4Rr)^2)+48s^2R^2r^2}{4sr(s^2-(2R+r)^2)}.$$

Solution. A proof for the identity from the statement was givenby M. Bencze in math journal Sclipirea Minţii – Vol. 7, No. 12, 2013.

PP.20679. In all nonisosceles triangle holds

$$\left(\sum \frac{\sin\frac{A-B}{2}\sin^2\frac{C}{2}}{\cos^2\frac{C}{2}}\right)\left(\sum \frac{\cos^2\frac{C}{2}}{\sin\frac{A-B}{2}\sin^2\frac{C}{2}}\right) = 5 - \frac{16R}{r} + \left(\frac{s}{r}\right)^2.$$

Solution. See the math journal Sclipirea Minţii – Vol. 7, No. 12, 2013.

PP.20680. In all nonisosceles triangle holds

$$\left(\sum \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}\right)\left(\sum \frac{\cos\frac{A}{2}}{\sin\frac{B-C}{2}}\right) = 1 - \frac{2r}{R}.$$

Solution 1. We have $\sin\frac{B-C}{2} = \frac{b-c}{a}\cos\frac{A}{2}$, so

$$\sum \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}} = \sum \frac{b-c}{a} = -\frac{(a-b)(b-c)(c-a)}{abc}, \text{ and}$$

$$\sum \frac{\cos\frac{A}{2}}{\sin\frac{B-C}{2}} = \sum \frac{a}{b-c} = \frac{\sum (a^2b + a^2c - a^3 - abc)}{(a-b)(b-c)(c-a)}.$$

Since

$$\sum (a^2b + a^2c - a^3 - abc) = \sum a^2(2s-a) - \sum a^3 - 3abc =$$
$$= 2s\sum a^2 - 2\left(\sum a^3 - 3abc\right) - 9abc =$$
$$= 4s(s^2 - r^2 - 4Rr) - 2(2s)(2s^2 - 2r^2 - 8Rr - s^2 - r^2 - 4Rr) - 36Rrs =$$
$$= 4s(s^2 - r^2 - 4Rr - s^2 + 3r^2 + 12Rr - 9Rr) = 4s(2r^2 - Rr).$$

Therefore,

$$\left(\sum \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}\right)\left(\sum \frac{\cos\frac{A}{2}}{\sin\frac{B-C}{2}}\right) = \frac{(a-b)(b-c)(c-a)}{abc} \cdot \frac{4rs(R-2r)}{(a-b)(b-c)(c-a)} =$$

$= \frac{R-2r}{R} = 1 - \frac{2r}{R}$, and the proof is complete.

Solution 2. See the math journal Sclipirea Minţii – Vol. 6, No. 11, 2013, p. 9.

PP.20681. In all nonisosceles triangle holds

$$\left(\sum\frac{\sin\frac{C-B}{2}tg\frac{A}{2}}{\sin\frac{C-B}{2}-\sin\frac{A}{2}}\right)\left(\sum\frac{\cos\frac{C-B}{2}-\sin\frac{A}{2}}{\sin\frac{C-B}{2}tg\frac{A}{2}}\right)=5-\frac{16R}{r}+\left(\frac{s}{r}\right)^2.$$

Solution. See the math journal Sclipirea Minţii – Vol. 6, No. 11, p. 9.

PP.20684. If $a,b,c>0$, then $4\le\left(\frac{a+b}{2a}+\frac{2a}{a+b}\right)\left(\frac{a+b}{2b}+\frac{2b}{a+b}\right)\le\left(\frac{a}{b}+\frac{b}{a}\right)^2$.

Solution. Because $\frac{a+b}{2a}+\frac{2a}{a+b}\ge 2$ and $\frac{a+b}{2b}+\frac{2b}{a+b}\ge 2$ the left inequality yields immediately. For the right inequality we have:

$$\frac{a+b}{2a}+\frac{2a}{a+b}\ge\frac{a}{b}+\frac{b}{a}\Leftrightarrow 2a^3-3a^2b+b^3\ge 0\Leftrightarrow(a-b)^2(2a+b)\ge 0,\text{ true.}$$

Similar, we have $\frac{a+b}{2b}+\frac{2b}{a+b}\ge\frac{a}{b}+\frac{b}{a}$, from where by multiplying yields the desired result.

PP.20690. Solve in R the equation $x^3-7x+7=0$.

Solution. The equation $x^3+px+q=0$, has three real roots if $\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3<0$.

In our case $p=-7, q=7$ and $\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3=\frac{49}{4}-\frac{343}{27}=-\frac{409}{108}$.

We have $r=\sqrt{\frac{-p^3}{27}}=\sqrt{\frac{343}{27}}=\frac{7}{3}\sqrt{\frac{7}{3}}$ and $\cos\varphi=\frac{-\frac{q}{2}}{\sqrt{\frac{-p^3}{27}}}=-\frac{3}{2}\sqrt{\frac{3}{7}}$.

The three roots are:

$$x_1=2\cdot\sqrt[3]{r}\cdot\cos\frac{\varphi}{3},\quad x_2=2\cdot\sqrt[3]{r}\cdot\cos\left(\frac{\varphi}{3}+120^0\right)\text{ and }x_3=2\cdot\sqrt[3]{r}\cdot\cos\left(\frac{\varphi}{3}+240^0\right),$$

and we are done.

PP.20705. If $a_k>0$ $(k=1,2,...n)$, then $\sum_{cyclic}\left(\frac{a_2+a_3}{a_1^2}+\frac{a_1+a_3}{a_2^2}+\frac{a_1+a_2}{a_3^2}\right)\ge\frac{6n^2}{\sum_{k=1}^{n}a_k}$.

Solution 1. We have the inequality:

$$\frac{a+b}{c^2}+\frac{b+c}{a^2}+\frac{c+a}{b^2}\geq\frac{18}{a+b+c} \qquad (1)$$

which is Problem L222 from Rec.Mat. 1/2012, proposed by Florin Stanescu.
Solutions, refinements and generalizations you can find in:
[1] D.M. Bătineţu-Giurgiu, Neculai Stanciu, I.V. Codreanu, Problema L222 din nr. 1/2012 revizitată, Rec.Mat. nr. 1/2013;
[2] Titu Zvonaru, Câteva soluţii la problema L222 din Rec.Mat. nr. 1/2012, Rec. Mat, nr. 2/2012.
By (1) and the inequality of Harald Bergström we obtain that:

$$\sum_{cyclic}\left(\frac{a_2+a_3}{a_1^2}+\frac{a_1+a_3}{a_2^2}+\frac{a_1+a_2}{a_3^2}\right)\geq\sum_{cyclic}\frac{18}{a_1+a_2+a_3}\geq 18\cdot\frac{n^2}{\sum\limits_{cyclic}a_1+a_2+a_3}=$$

$$=18\cdot\frac{n^2}{3\sum\limits_{k=1}^{n}a_k}=\frac{6n^2}{\sum\limits_{k=1}^{n}a_k},$$

and first solution is complete.

Solution 2. We prove that: $\frac{a+b}{c^2}+\frac{b+c}{a^2}+\frac{c+a}{b^2}\geq\frac{2}{a}+\frac{2}{b}+\frac{2}{c}$ (2)

For (2) we give also two solutions.

(i) $\sum\frac{a+b}{c^2}-2\sum\frac{1}{a}=\sum\left(\frac{a+b}{c^2}-\frac{2}{c}\right)=\sum\left(\frac{a-c}{c^2}+\frac{b-c}{c^2}\right)=$

$$=\sum\frac{a-c}{c^2}+\sum\frac{b-c}{c^2}=\sum\frac{a-c}{c^2}+\sum\frac{c-a}{a^2}=\sum\frac{(c-a)(c^2-a^2)}{c}\geq 0.$$

(ii) $\frac{a}{c^2}+\frac{1}{a}\geq\frac{2}{c}$; $\frac{a}{b^2}+\frac{1}{a}\geq\frac{2}{b}$; $\frac{b}{c^2}+\frac{1}{b}\geq\frac{2}{c}$; $\frac{b}{a^2}+\frac{1}{b}\geq\frac{2}{a}$; $\frac{c}{a^2}+\frac{1}{c}\geq\frac{2}{a}$; $\frac{c}{b^2}+\frac{1}{c}\geq\frac{2}{b}$ which by adding yields (2).
By (2) and the inequality of Harald Bergström we obtain that:

$$\sum_{cyclic}\left(\frac{a_2+a_3}{a_1^2}+\frac{a_1+a_3}{a_2^2}+\frac{a_1+a_2}{a_3^2}\right)\geq 2\sum_{cyclic}\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)\geq$$

$$\geq 2\cdot\frac{n^2}{\sum\limits_{cyclic}a_1}+2\cdot\frac{n^2}{\sum\limits_{cyclic}a_2}+2\cdot\frac{n^2}{\sum\limits_{cyclic}a_3}=\frac{6n^2}{\sum\limits_{k=1}^{n}a_k},$$

and we are done.

2. Other solutions from some problems from School Science and Mathematics journal

By Nela Ciceu, Roşiori, Bacău
and Roxana Mihaela Stanciu, Buzău

- **5307:** *Proposed by Haishen Yao and Howard Sporn, Queensborough Community College, Bayside, NY*

  Solve for x:

$$\sqrt{x^{15}} = \sqrt{x^{10}-1} + \sqrt{x^5-1}.$$

Solution:

We denote $x^5 = y$, and after squaring we obtain

$$y^3 - y^2 - y + 2 = 2\sqrt{(y^2-1)(y-1)}$$, and squaring again we obtain

$$y^2(y^4 - 2y^3 - y^2 + 2y + 1) = 0 \Leftrightarrow y^2(y^2 - y - 1)^2 = 0$$, which yields that

$$y = 0,\ y = \frac{1+\sqrt{5}}{2},\ y = \frac{1-\sqrt{5}}{2}.$$

Therefore we have to solve in complex number the equations

$$x^5 = 0,\ x^5 = \frac{1+\sqrt{5}}{2} \text{ and } x^5 = \frac{1-\sqrt{5}}{2}.$$

We obtain the solutions

$$x = 0,\ x_k = \sqrt[5]{\frac{1+\sqrt{5}}{2}}\left(\cos\frac{2k\pi}{5} + i\sin\frac{2k\pi}{5}\right),\ k = 0,1,2,3,4 \text{ and}$$

$$x_m = \sqrt[5]{\frac{1-\sqrt{5}}{2}}\left(\cos\frac{2m\pi}{5} + i\sin\frac{2m\pi}{5}\right),\ m = 0,1,2,3,4.$$

- **5308:** *Proposed by Kenneth Korbin, New York, NY*

  Given the sequence
  $$t = (1, 7, 41, 239, \ldots)$$
  with $t_n = 6t_{n-1} - t_{n-2}$. Let (x, y, z) be a triple of consecutive terms in this sequence with $x < y < z$.

  Part 1) Express the value of x in terms of y and express the value of y in terms of x.

  Part 2) Express the value of x in terms of z and express the value of z in terms of x.

Solution:

We have the equation of recurrence $r^2 - 6r + 1 = 0$, whith $r_{1,2} = 3 \pm 2\sqrt{2}$.

Yields $t_n = a(3+2\sqrt{2})^n + b(3-2\sqrt{2})^n$. By $t_1 = 1$ and $t_2 = 7$ we deduce

$a = \dfrac{\sqrt{2}-1}{2}, b = -\dfrac{\sqrt{2}+1}{2}$. We obtain $t_n = \dfrac{\sqrt{2}-1}{2}(3+2\sqrt{2})^n - \dfrac{\sqrt{2}+1}{2}(3-2\sqrt{2})^n$.

1)Since $(\sqrt{2}-1)(3+2\sqrt{2}) = \sqrt{2}+1$ and $(\sqrt{2}+1)(3-2\sqrt{2}) = \sqrt{2}-1$, we have

$$x = \frac{\sqrt{2}-1}{2}(3+2\sqrt{2})^n - \frac{\sqrt{2}+1}{2}(3-2\sqrt{2})^n$$

$y = \dfrac{\sqrt{2}+1}{2}(3+2\sqrt{2})^n - \dfrac{\sqrt{2}-1}{2}(3-2\sqrt{2})^n$, which yields that

$$x(\sqrt{2}+1) - y(\sqrt{2}-1) = -2\sqrt{2}(3-2\sqrt{2})^n$$

$x(\sqrt{2}-1) - y(\sqrt{2}+1) = -2\sqrt{2}(3+2\sqrt{2})^n$, which by multiplying and taking into account that

$$(3-2\sqrt{2})(3+2\sqrt{2}) = 1 \text{ yields}$$
$$x^2 + y^2 - 6xy = 8,$$

from where we obtain the value of x in terms of y and the express of y in terms of x.

Because $x < y$ we obtain $x = 3y - \sqrt{8y^2 + 8}$ and $y = 3x + \sqrt{8x^2 + 8}$.

2) As above, we have

$$x^2 + y^2 - 6xy = 8$$
$$y^2 + z^2 - 6yz = 8$$

and then

$y = 3z - \sqrt{8z^2 + 8}$, so

$$x = 3\left(3z - \sqrt{8z^2+8}\right) - \sqrt{8\left(3z - \sqrt{8z^2+8}\right)^2 + 8}$$

$z = 3y + \sqrt{8y^2 + 8}$, so

$$z = 3\left(3x + \sqrt{8x^2+8}\right) + \sqrt{8\left(3x + \sqrt{8x^2+8}\right)^2 + 8}.$$

- **5309:** *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA*

  Consider the expression $3^n + n^2$ for positive integers n. It is divisible by 13 for $n = 18$ and $n = 19$. Prove, however, that it is never divisible by 13 for three consecutive values of n.

Solution:

The values modulo 13 of n^2 for 13 consecutive values of n are:

0,1, 4, 9, 3, 12, 10, 10, 12, 3, 9, 4,1.

Since $3^3 \equiv 1 \pmod{13}$ yields that $3^n \pmod{13}$ it can take only the values 1, 3, 9.

The expressions $3^n + n^2$ and $3^{n+1} + (n+1)^2$ are simultaneously divisible by 13 only if $n^2 \equiv 12 \pmod{13}$ and $(n+1)^2 \equiv 10 \pmod{13}$, but then $(n+2)^2 \equiv 10 \pmod{13}$ which added with $3^{n+2} \equiv 9 \pmod{13}$ does not give $0 \pmod{13}$.

3. EXERCIŢII CU PROGRESII ARITMETICE.GENERALIZĂRI.

Prof. Ciobîcă C. Constantin - COLEGIUL „ VASILE LOVINESCU”, FĂLTICENI
Prof. Ciobîcă Elena - COLEGIU TEHNIC MIHAI BĂCESCU; FĂLTICENI

1. Fie $(a_n)_{n\in N^*}$ un şir de numere reale în progresie aritmetică. Demonstraţi egalitatea:

$$\frac{1}{\sum_{k=1}^{n} a_{3k+2}\cdot\sum_{k=1}^{n} a_{3k+5}}+\frac{1}{\sum_{k=1}^{n} a_{3k+5}\cdot\sum_{k=1}^{n} a_{3k+8}}+\ldots+\frac{1}{\sum_{k=1}^{n} a_{3k+3i+2}\cdot\sum_{k=1}^{n} a_{3k+3i+5}}=$$

$$=\frac{1+i}{\sum_{k=1}^{n} a_{3k+2}\cdot\sum_{k=1}^{n} a_{3k+3i+5}};\ \forall i\in N, \forall n\in N^*$$

Rezolvare:

$$\sum_{k=1}^{n} a_{3k+2}=a_5+a_8+\ldots.+a_{3n+8}$$

$$\sum_{k=1}^{n} a_{3k+5}=a_8+a_{11}+\ldots+a_{3n+5}$$

$$\sum_{k=1}^{n} a_{3k+5}-\sum_{k=1}^{n} a_{3k+2}=a_{3n+5}-a_5=a_1+(3n+4)r-a_1-4r=3nr$$

$$\sum_{k=1}^{n} a_{3k+3i+2}=a_{5+3i}+a_{8+3i}+\ldots+a_{3n+3i+2}$$

$$\sum_{k=1}^{n} a_{3k+3i+5}=a_{8+3i}+a_{11+3i}+\ldots+a_{3n+3i+5}$$

$$\sum_{k=1}^{n} a_{3k+3i+5}-\sum_{k=1}^{n} a_{3k+3i+2}=a_{3n+3i+5}-a_{5+3i}=a_1+(3n+3i+4)r-a_1-(3i+4)r=3nr$$

$$\frac{1}{\sum_{k=1}^{n} a_{3k+2}\cdot\sum_{k=1}^{n} a_{3k+5}}+\frac{1}{\sum_{k=1}^{n} a_{3k+5}\cdot\sum_{k=1}^{n} a_{3k+8}}+\ldots+\frac{1}{\sum_{k=1}^{n} a_{3k+3i+2}\cdot\sum_{k=1}^{n} a_{3k+3i+5}}=$$

$$=\frac{1}{3nr}\left(\frac{1}{\sum_{k=1}^{n} a_{3k+2}}-\frac{1}{\sum_{k=1}^{n} a_{3k+3i+5}}\right)$$

$$\sum_{k=1}^{n} a_{3k+3i+5} = a_{8+3i} + a_{11+3i} + \ldots + a_{3n+3i+5}$$

$$\sum_{k=1}^{n} a_{3k+2} = a_5 + a_8 + \ldots. + a_{3n+8}$$

$$\sum_{k=1}^{n} a_{3k+3i+5} - \sum_{k=1}^{n} a_{3k+2} = (a_{8+3i} - a_5) + (a_{11+3i} - a_8) + \ldots + (a_{3n+3i+5} - a_{3n+2}) =$$

$$= \underbrace{(3+3i)r + (3+3i)r + \ldots + (3+3i)r}_{de \quad n \quad ori} = 3nr(i+1)$$

$$= \frac{1}{3nr}\left(\frac{1}{\sum_{k=1}^{n} a_{3k+2}} - \frac{1}{\sum_{k=1}^{n} a_{3k+3i+5}} \right) = \frac{1}{3nr} \cdot \frac{3nr(i+1)}{\sum_{k=1}^{n} a_{3k+2} \cdot \sum_{k=1}^{n} a_{3k+3i+5}} = \frac{1+i}{\sum_{k=1}^{n} a_{3k+2} \cdot \sum_{k=1}^{n} a_{3k+3i+5}}$$

2. Fie $(a_n)_{n \in N^*}$ un şir de numere reale în progresie aritmetică. Demonstraţi egalitatea:

$$\frac{1}{\sum_{k=1}^{n}(a_{4k+3} + a_{7k+2}) \cdot \sum_{k=1}^{n}(a_{4k+7} + a_{7k+9})} + \frac{1}{\sum_{k=1}^{n}(a_{4k+7} + a_{7k+9}) \cdot \sum_{k=1}^{n}(a_{4k+11} + a_{7k+16})} + \ldots$$

$$\ldots. + \frac{1}{\sum_{k=1}^{n}(a_{4k+4i+3} + a_{7k+7i+2}) \cdot \sum_{k=1}^{n}(a_{4k+4i+7} + a_{7k+7i+9})} =$$

$$= \frac{i+1}{\sum_{k=1}^{n}(a_{4k+3} + a_{7k+2}) \cdot \sum_{k=1}^{n}(a_{4k+4i+3} + a_{7k+7i+9})}, \quad \forall i \in N; \forall n \in N^*$$

Rezolvare:

$$\sum_{k=1}^{n}(a_{4k+3} + a_{7k+2}) = \sum_{k=1}^{n} a_{4k+3} + \sum_{k=1}^{n} a_{7k+2} = (a_7 + a_{11} + \ldots + a_{4n+3}) + (a_9 + a_{16} + \ldots + a_{7n+2})$$

$$\sum_{k=1}^{n}(a_{4k+7} + a_{7k+9}) = \sum_{k=1}^{n} a_{4k+7} + \sum_{k=1}^{n} a_{7k+9} = (a_{11} + a_{15} + \ldots + a_{4n+7}) + (a_{16} + a_{23} + \ldots + a_{7n+9})$$

$$\sum_{k=1}^{n}(a_{4k+7} + a_{7k+9}) - \sum_{k=1}^{n}(a_{4k+3} + a_{7k+2}) = (a_{4n+7} - a_7) + (a_{7n+9} - a_9) =$$

$$= a_1 + (4n+6)r - a_1 - 6r + a_1 + (7n+8)r - a_1 - 8r = 4nr + 7nr = 11nr$$

$$\frac{1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+7}+a_{7k+9}\right)}=\frac{1}{11nr}\cdot\left(\frac{1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)}-\frac{1}{\sum_{k=1}^{n}\left(a_{4k+7}+a_{7k+9}\right)}\right)$$

.

$$\sum_{k=1}^{n}\left(a_{4k+4i+3}+a_{7k+7i+2}\right)=\sum_{k=1}^{n}a_{4k+4i+3}+\sum_{k=1}^{n}a_{7k+7i+2}=$$

$$=\left(a_{4i+7}+a_{4i+11}+...+a_{4n+4i+3}\right)+\left(a_{7i+9}+a_{7i+16}+....+a_{7n+7i+2}\right)$$

$$\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)=\sum_{k=1}^{n}a_{4k+4i+7}+\sum_{k=1}^{n}a_{7k+7i+9}=$$

$$=\left(a_{4i+11}+a_{4i+15}+...+a_{4n+4i+7}\right)+\left(a_{7i+16}+a_{7i+23}+....+a_{7n+7i+9}\right)$$

$$\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)-\sum_{k=1}^{n}\left(a_{4k+4i+3}+a_{7k+7i+2}\right)=\left(a_{4n+4i+7}-a_{4i+7}\right)+\left(a_{7n+7i+9}-a_{7i+9}\right)=$$

$$=a_1+(4n+4i+6)r-a_1-(4i+6)r+a_1+(7n+7i+8)r-a_1-(7i+8)r=$$

$$=4nr+7nr=11nr$$

$$\frac{1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+7}+a_{7k+9}\right)}+\frac{1}{\sum_{k=1}^{n}\left(a_{4k+7}+a_{7k+9}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+11}+a_{7k+16}\right)}+...$$

$$....+\frac{1}{\sum_{k=1}^{n}\left(a_{4k+4i+3}+a_{7k+7i+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)}=$$

$$=\frac{1}{11nr}\left[\frac{1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)}-\frac{1}{\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)}\right]$$

$$\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)-\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)=\left(a_{4i+11}+a_{4i+15}+...+a_{4n+4i+7}\right)+$$

$$+\left(a_{7i+16}+a_{7i+23}+...+a_{7n+7i+9}\right)-\left(a_7+a_{11}+...+a_{4n+3}\right)-\left(a_9+a_{16}+...+a_{7n+2}\right)=$$

$$=a_1+(4i+10)r-a_1-6r+a_1+(4i+14)r-a_1-10r+....$$

$$+a_1+(7i+15)r-a_1-8r+a_1+(7i+22)r-a_1-15r+...=$$

$$=\underbrace{(4i+4)r+(4i+4)r+...}_{de\ \ n\ \ ori}+\underbrace{(7i+7)r+(7i+7)r+...}_{de\ \ n\ \ ori}=4nr(i+1)+7nr(i+1)=11nr(i+1)$$

$$\frac{1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+7}+a_{7k+9}\right)}+\frac{1}{\sum_{k=1}^{n}\left(a_{4k+7}+a_{7k+9}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+11}+a_{7k+16}\right)}+...$$

$$....+\frac{1}{\sum_{k=1}^{n}\left(a_{4k+4i+3}+a_{7k+7i+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)}=$$

$$=\frac{1}{11nr}\left[\frac{1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)}-\frac{1}{\sum_{k=1}^{n}\left(a_{4k+4i+7}+a_{7k+7i+9}\right)}\right]=$$

$$=\frac{1}{11nr}\cdot\frac{11nr(i+1)}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+4i+3}+a_{7k+7i+9}\right)}=$$

$$=\frac{i+1}{\sum_{k=1}^{n}\left(a_{4k+3}+a_{7k+2}\right)\cdot\sum_{k=1}^{n}\left(a_{4k+4i+3}+a_{7k+7i+9}\right)}$$

3. Fie $(a_n)_{n\in N^*}$ un şir de numere reale în progresie aritmetică. Demonstraţi egalitatea:

$$\frac{1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)}+$$

$$+\frac{1}{\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+11}+a_{6k+13}+a_{7k+15}\right)}+...$$

$$...+\frac{1}{\sum_{k=1}^{n}\left(a_{5k+5i+1}+a_{6k+6i+1}+a_{7k+7i+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)}=$$

$$=\frac{i+1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)};\forall i\in N;\forall n\in N^*$$

Rezolvare:

$$\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)=\sum_{k=1}^{n}a_{5k+1}+\sum_{k=1}^{n}a_{6k+1}+\sum_{k=1}^{n}a_{7k+1}=$$

$$=\left(a_6+a_{11}+....+a_{5n+1}\right)+\left(a_7+a_{13}+....+a_{6n+1}\right)+\left(a_8+a_{15}+....+a_{7n+1}\right)$$

$$\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)=\sum_{k=1}^{n}a_{5k+6}+\sum_{k=1}^{n}a_{6k+7}+\sum_{k=1}^{n}a_{7k+8}=$$

$$=\left(a_{11}+a_{16}+....+a_{5n+6}\right)+\left(a_{13}+a_{19}+....+a_{6n+7}\right)+\left(a_{15}+a_{22}+....+a_{7n+8}\right)$$

$$\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)-\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)=\left(a_{5n+6}-a_6\right)+\left(a_{6n+7}-a_7\right)+$$

$$+\left(a_{7n+8}-a_8\right)=5nr+6nr+7nr=18nr$$

$$\frac{1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)}=$$

$$=\frac{1}{18nr}\cdot\left[\frac{1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)}-\frac{1}{\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)}\right]$$

$$\sum_{k=1}^{n}\left(a_{5k+5i+1}+a_{6k+6i+1}+a_{7k+7i+1}\right)=\sum_{k=1}^{n}a_{5k+5i+1}+\sum_{k=1}^{n}a_{6k+6i+1}+\sum_{k=1}^{n}a_{7k+7i+1}=$$
$$=\left(a_{5i+6}+a_{5i+11}+...+a_{5n+5i+1}\right)+\left(a_{6i+7}+a_{6i+13}+...+a_{6n+6i+1}\right)+$$
$$+\left(a_{7i+8}+a_{7i+15}+....+a_{7n+7i+1}\right)$$

$$\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)=\sum_{k=1}^{n}a_{5k+5i+6}+\sum_{k=1}^{n}a_{6k+6i+7}+\sum_{k=1}^{n}a_{7k+7i+8}=$$
$$=\left(a_{5i+11}+a_{5i+16}+...+a_{5n+5i+6}\right)+\left(a_{6i+13}+a_{6i+19}+...+a_{6n+6i+7}\right)+$$
$$+\left(a_{7i+15}+a_{7i+22}+....+a_{7n+7i+8}\right)$$

$$\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)-\sum_{k=1}^{n}\left(a_{5k+5i+1}+a_{6k+6i+1}+a_{7k+7i+1}\right)=$$
$$=\left(a_{5n+5i+6}-a_{5i+6}\right)+\left(a_{6i+6n+7}-a_{6i+7}\right)+\left(a_{7n+7i+8}-a_{7i+8}\right)=$$
$$=5nr+6nr+7nr=18nr$$

$$\frac{1}{\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+5i+1}+a_{6k+6i+1}+a_{7k+7i+1}\right)}=$$

$$=\frac{1}{18nr}\cdot\left[\frac{1}{\sum_{k=1}^{n}\left(a_{5k+5i+1}+a_{6k+6i+1}+a_{7k+7i+1}\right)}-\frac{1}{\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)}\right]$$

$$\frac{1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)}+$$
$$+\frac{1}{\sum_{k=1}^{n}\left(a_{5k+6}+a_{6k+7}+a_{7k+8}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+11}+a_{6k+13}+a_{7k+15}\right)}+...$$
$$...+\frac{1}{\sum_{k=1}^{n}\left(a_{5k+5i+1}+a_{6k+6i+1}+a_{7k+7i+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)}=$$

$$=\frac{1}{18nr}\cdot\left[\frac{1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)}-\frac{1}{\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)}\right]$$

$$\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)-\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)=$$

$$=\sum_{k=1}^{n}\left(a_{5k+5i+6}-a_{5k+1}\right)+\sum_{k=1}^{n}\left(a_{6k+6i+7}-a_{6k+1}\right)+\sum_{k=1}^{n}\left(a_{7k+7i+8}-a_{7k+1}\right)=$$

$$=\sum_{k=1}^{n}(5i+5)r+\sum_{k=1}^{n}(6i+6)r+\sum_{k=1}^{n}(7i+7)r=18nr(i+1)$$

$$\Rightarrow S=\frac{i+1}{\sum_{k=1}^{n}\left(a_{5k+1}+a_{6k+1}+a_{7k+1}\right)\cdot\sum_{k=1}^{n}\left(a_{5k+5i+6}+a_{6k+6i+7}+a_{7k+7i+8}\right)};\ \forall i\in N;\forall n\in N^{*}$$

4. Fie $(a_n)_{n\in N^*}$ un şir de numere reale în progresie aritmetică. Demonstraţi egalitatea:

$$\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)\cdot\sum_{k=}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)}+\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+20+i}\right)}+$$

$$+\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}=\frac{p+1}{\sum_{k=}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)};$$

$\forall p\in N;\forall n\in N^{*};\forall m\in N^{*}$

Rezolvare:

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)=\sum_{k=1}^{n}\left(a_{10k+1}+a_{10k+2}+\ldots+a_{10k+m}\right)=\sum_{k=1}^{n}a_{10k+1}+\sum_{k=1}^{n}a_{10k+2}+\ldots+\sum_{k=1}^{n}a_{10k+m}$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i+10}\right)=\sum_{k=1}^{n}\left(a_{10k+11}+a_{10k+12}+\ldots+a_{10k+m+10}\right)=\sum_{k=1}^{n}a_{10k+11}+\sum_{k=1}^{n}a_{10k+12}+\ldots+\sum_{k=1}^{n}a_{10k+m+10}$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)-\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)=\sum_{k=1}^{n}\left(a_{10k+11}-a_{10k+1}\right)+\sum_{k=1}^{n}\left(a_{10k+12}-a_{10k+2}\right)+\ldots+$$

$$+\sum_{k=1}^{n}\left(a_{10k+10+m}-a_{10k+m}\right)=\underbrace{\sum_{k=1}^{n}10r+\sum_{k=1}^{n}10r+\ldots+\sum_{k=1}^{n}10r}_{de\ \ m\ \ ori}=10nmr$$

$$\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)}=\frac{1}{10nmr}\left[\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)}-\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)}\right]$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i}\right)=\sum_{k=1}^{n}\left(a_{10k+10p+1}+a_{10k+10p+2}+....+a_{10k+10p+m}\right)=$$

$$=\sum_{k=1}^{n}a_{10k+10p+1}+\sum_{k=1}^{n}a_{10k+10p+2}+...+\sum_{k=1}^{n}a_{10k+10p+m}$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i+10}\right)=\sum_{k=1}^{n}\left(a_{10k+10p+11}+a_{10k+10p+12}+...+a_{10k+10p+10+m}\right)=$$

$$=\sum_{k=1}^{n}a_{10k+10p+11}+\sum_{k=1}^{n}a_{10k+10p+12}+...+\sum_{k=1}^{n}a_{10k+10p+10+m}$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i+10}\right)-\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i}\right)=\sum_{k=1}^{n}\left(a_{10k+10p+11}-a_{10p+10k+1}\right)+$$

$$+\sum_{k=1}^{n}\left(a_{10k+10p+12}-a_{10p+10k+2}\right)+...+\sum_{k=1}^{n}\left(a_{10k+10p+10+m}-a_{10k+10p+m}\right)=$$

$$=\underbrace{\sum_{k=1}^{n}10r+\sum_{k=1}^{n}10r+...+\sum_{k=1}^{n}10r}_{de\quad m\quad ori}=10nmr$$

$$\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}=\frac{1}{10nmr}\left[\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i}\right)}-\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}\right]$$

$$\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)\cdot\sum_{k=}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)}+\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+20+i}\right)}+$$

$$+\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}=$$

$$=\frac{1}{10nmr}\cdot\left[\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)}-\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}\right]$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)-\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)=\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10p+10k+10+i}-\sum_{i=1}^{m}a_{10k+i}\right)=$$

$$=\sum_{k=1}^{n}\left[\sum_{i=1}^{m}\left(a_{10k+10p+10+i}-a_{10k+i}\right)\right]=\sum_{k=1}^{n}\left[\sum_{i=1}^{m}(10p+10)r\right]=\sum_{k=1}^{n}(10p+10)rm=10(p+1)nmr$$

$$\frac{1}{10nmr}\cdot\left[\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)}-\frac{1}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}\right]=$$

$$=\frac{1}{10nmr}\cdot\frac{10nmr(p+1)}{\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)}=\frac{p+1}{\sum_{k=}^{n}\left(\sum_{i=1}^{m}a_{10k+i}\right)\cdot\sum_{k=1}^{n}\left(\sum_{i=1}^{m}a_{10k+10p+10+i}\right)};$$

4. Metode de integrare numerică: polinomul de interpolare Lagrange, formula lui Simpson, aplicație.

Prof. Boer Elena Milena, Școala Gimnazială Vulcan, Jud. Brașov

1. Polinomul de interpolare Lagrange.

Considerăm o funcție y=f(x) pe care dorim să o interpolăm. Pentru aceasta, presupunem cunoscute valorile $y_1, y_2, \dots, y_n,$ corespunzătoare argumentelor $x_1, x_2, \dots, x_n$.

Construim polinomul $P_m(x)$ care are în punctele $x_i,$ aceleași valori ca și funcția f(x).

$$P_m(x_i) = y_i, \; i = \overline{1,n} \qquad (1)$$

Fie familia de polinoame $p_i(x_j) = \delta_{ij}$ (2)

δ_{ij} reprezintă simbolul lui Kronecker. $p_i(x)$ se anulează în toate punctele mai puțin x_i. Îl putem scrie pe $p_i(x)$ astfel:

$$p_i(x) = a_i \prod_{j\neq i}^{n} (x - x_j) \qquad (3)$$

$p_i(x)$ – este un polinom de ordin n-1.

Deoarece $p_i(x_i) = 1$, coeficientul a_i este:

$$a_i = \frac{1}{\prod_{j\neq i}^{n}(x_i - x_j)}$$, ceea ce înseamnă că

$$p_i(x) = \frac{\prod_{j\neq i}^{n}(x - x_j)}{\prod_{j\neq i}^{n}(x_i - x_j)} \qquad (4)$$

Polinomul $P_m(x),$ unde m = n-1, îl putem scrie ca o combinație liniară a polinoamelor $p_i(x)$.

$$P_{n-1}(x) = \sum_{i=1}^{n} p_i(x) y_i \qquad (5)$$

Observăm că $P_{n-1}(x_1) = y_1, P_{n-1}(x_2) = y_2, \dots, P_{n-1}(x_n) = y_n$.

Înlocuindrelația (4) a polinoamelor $p_i(x)$, se obține polinomul de interpolare al lui Lagrange:

$$P_{n-1}(x) = \sum_{i=1}^{n} \frac{\prod_{j\neq i}^{n}(x - x_j)}{\prod_{j\neq i}^{n}(x_i - x_j)} y_i \qquad (6)$$

2. Formula lui Simpson.

Fie $I = \int_a^b f(x),$ integrală definită pe intervalul [a,b],împărțitîntr-un număr de n-1 subintervale de lungime $h = \frac{b-a}{n-1}$, prin punctele $x_i = a + (i-1)h$. Se presupun cunoscute valorile funcției f(x) în punctele $x_1, x_2, \dots, x_n$. Aproximăm funcția f(x) prin polinomul de interpolare Lagrange.

$$P_{n-1}(x) = \sum_{i=1}^{n} \frac{\prod_{j\neq i}^{n}(x-x_j)}{\prod_{j\neq i}^{n}(x_i-x_j)} f_i$$

Introducem mărimea adimensională $q = \frac{x-a}{h}$ și atunci vom avea:

$$\prod_{j\neq i}^{n}(x-x_j) = h^{n-1}\prod_{j\neq i}^{n}[q-(j-1)]$$

$$\prod_{j\neq i}^{n}(x_i-x_j) = (-1)^{n-1}h^{n-1}(i-1)!\,(n-1)!$$

Astfel pentru polinomul de interpolare Lagrange se obține relația:

$$P_{n-1}(x) = \sum_{i=1}^{n} \frac{\prod_{j\neq i}^{n}[q-(j-1)]}{(-1)^{n-1}(i-1)!(n-1)!} f_i \qquad (7)$$

Rezultă astfel, următoarea aproximație pentru integrala dată:

$$\int_a^b f(x) \cong \int_a^b P_{n-1}(x)dx = \sum_{i=1}^{n} A_i f_i \qquad (8)$$

unde

$$A_i = \int_a^b \frac{\prod_{j\neq i}^{n}[q-(j-1)]}{(-1)^{n-1}(i-1)!(n-1)!}dx = \frac{h\int_0^{n-1}\prod_{j\neq i}^{n}[q-(j-1)]dq}{(-1)^{n-1}(i-1)!(n-1)!}$$

Notăm $A_i = (b-a)H_i$. Din relația (8) obținem formula de cuadratură Newton – Cotes:

$$\int_a^b f(x) \cong (b-a)\sum_{i=1}^{n} H_i f_i \qquad (9)$$

Astfel pentru n=3 se obțin relațiile:

$$H_1 = \frac{1}{4}\int_0^2 (q-1)(q-2)dq = \frac{1}{6}; \quad H_2 = -\frac{1}{2}\int_0^2 q(q-2)dq = \frac{2}{3};$$

$$H_3 = \frac{1}{4}\int_0^2 q(q-1)dq = \frac{1}{6};$$

Rezultă formula lui Simpson:

$$\int_{x_1}^{x_3} f(x)dx \approx \frac{h}{3}(f_1 + 4f_2 + f_3) \qquad (10)$$

Pentru o mai bună acuratețe, generalizăm relația (10) și împărțim intervalul [a,b] printr-un număr impar, n=2m+1 de puncte echidistante $x_i = a + \frac{i-1}{h}$. Aplicăm formula lui Simpson pentru fiecare din cele m subintervaleduble de lungime 2h: $[x_1, x_3]$, $[x_3, x_5]$, …, $[x_{n-2}, x_n]$, șiastfelintegraladefinităpeintervalul $[a, b]$ se poate scrie:

$$\int_a^b f(x)dx \approx \frac{h}{3}(f_1 + 4f_2 + f_3) + \frac{h}{3}(f_3 + 4f_4 + f_5) + \cdots + \frac{h}{3}(f_{n-2} + 4f_{n-1} + f_n)$$

Regrupândtermenii, se obține formula lui Simpson generalizată.

$$\int_a^b f(x)dx \approx \frac{h}{3}(f_1 + 4\theta_2 + 2\theta_1 + f_n) \qquad (11)$$

Unde:

$$\theta_1 = \sum_{i=1}^{(n-3)/2} f_{2i+1} \qquad (12)$$

$$\theta_2 = \sum_{i=1}^{(n-1)/2} f_{2i} \qquad (13)$$

3. Aplicație.

În încheierea acestui articol, prezint un program realizat în C, care calculează funcția de eroare prin integrare numerică, bazată pe formula lui Simpson.

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

```
#include <stdio.h>
#include <math.h>

float Func(float x)
{
 return exp(-pow(x,2));
}
float Simpson(float Func(float), float x)
{
 float h, h2, s1, s2, y;
 int i;
 const float pi = 4.0 * atan(1.0);
 int n = 51;

 h = x/(n-1); h2 = 2*h;
 s1 = 0.0; s2 = Func(h);

 for (i=1; i<=(n-3)/2; i++) {
  y = i*h2;
  s1 += Func(y); s2 += Func(y+h);
 }
 return 2*(h/3)*(1 + 4*s2 + 2*s1 + Func(x))/sqrt(pi);
}
int main()
{
 float x;
 int n;
   printf("x=");
   scanf("%f", &x);
   getchar;
   printf("\nerf(%.2f)=%2.6f\n", x, Simpson(Func, x));
   getchar();
   return 0;
}
```

Bibliografie:

1. Titus Beu, “Calcul numeric în C”, EdituraAlbastră, 2004.
2. S. Dumitru, A. Armășelu, A. Boer, “Fizicăprobabilistă – îndrumar de laborator”,
EdituraUniversității Transilvania, Brașov, 2004.